Solutions to Math 53 Midterm #2, 4/10/07

1

Write the integral either as a triple integral (cylindrical coordinates would be the easiest) or as a double integral (polar coordinates would be the easiest).

$$\int_0^{2\pi} \int_0^2 \int_{2r^2}^{12-r^2} r \, \mathrm{d}z \, \mathrm{d}r \, \mathrm{d}\theta = 24\pi$$

2

Maximum at $f(0, 2) = 14$.
Minimum at $f(0, -2) = -6$.

3

Change the order of integration and then evaluate:

$$\int_0^1 \int_x^1 \frac{\cos(y)}{y} \, \mathrm{d}y \, \mathrm{d}x = \int_0^1 \int_0^y \frac{\cos(y)}{y} \, \mathrm{d}x \, \mathrm{d}y = \sin(1)$$

4

Write the bounds in spherical coordinates and evaluate:

$$\iiint_E (x^2 + y^2 + z^2)^{3/2} \, \mathrm{d}V = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^1 (\rho^2)^{3/2} \rho^2 \sin\phi \, \mathrm{d}\rho \, \mathrm{d}\phi \, \mathrm{d}\theta = \frac{\pi}{3\sqrt{2}}$$

5

Make the suggested change of coordinates $x = u^2$, $y = v^2$. Under this transformation, the region R given in the problem transforms into S, the quarter unit circle in the first quadrant in the $u - v$ plane. The Jacobian is $4uv$.

$$\iint_R \frac{1}{\sqrt{xy}} \, \mathrm{d}x \, \mathrm{d}y = \iint_S \frac{1}{\sqrt{u^2 v^2}} 4uv \, \mathrm{d}u \, \mathrm{d}v = \iint_S 4 \, \mathrm{d}A = \pi$$

(For the last equality, we use the fact that $\iint_S 4 \, \mathrm{d}A$ denotes four times the area of the region S, the quarter unit circle, i.e. $4 \times \pi/4 = \pi$.)

6

Change into polar coordinates and then evaluate:

$$\int_0^{1/\sqrt{2}} \int_x^{\sqrt{1-x^2}} e^{x^2+y^2} \, \mathrm{d}y \, \mathrm{d}x = \int_{\pi/4}^{\pi/2} \int_0^1 e^{r^2} r \, \mathrm{d}r \, \mathrm{d}\theta = \frac{\pi}{8}(e - 1)$$