Solutions to Math 53 Midterm #2, 11/13/03

1

Maximum at $f\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 3 + 2\sqrt{2}$
Minimum at $f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 3 - 2\sqrt{2}$

2

Change the order of integration before evaluating:

$$\int_0^1 \int_{x^{2/3}}^1 x\cos(y^4)\,\mathrm{d}y\,\mathrm{d}x = \int_0^1 \int_0^{y3/2} x\cos(y^4)\,\mathrm{d}x\,\mathrm{d}y = \frac{\sin(1)}{8}$$

3

Change into polar coordinates before evaluating:

$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)^{2003}\,\mathrm{d}x\,\mathrm{d}x = \int_0^{\pi/2} \int_0^1 (r^2)^{2003} r\,\mathrm{d}r\,\mathrm{d}\theta = \frac{\pi}{8016}$$

4

Using the hint, make the change of coordinates $x = u + v\sqrt{3}$, $y = u - v\sqrt{3}$.
The original curve is transformed to $(u+v\sqrt{3})^2 + (u+v\sqrt{3})(u-v\sqrt{3}) + (u-v\sqrt{3})^2 = 1$, which after simplification becomes $u^2 + v^2 = 1/3$.
Evaluate the integral with the change of coordinates (further change into polar coordinates for ease of evaluation):

$$\iint_R 1\,\mathrm{d}x\,\mathrm{d}y = \iint_S 1\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,\mathrm{d}u\,\mathrm{d}v = \int_0^{2\pi} \int_0^{1/\sqrt{3}} 1 \cdot 2\sqrt{3} \cdot r\,\mathrm{d}r\,\mathrm{d}\theta = \frac{2\sqrt{3}}{3}\pi$$

(In the above, R is the region enclosed by $x^2 + xy + y^2 = 1$ in the $x - y$ plane, and S is the region enclosed by $u^2 + v^2 = 1/3$ in the $u - v$ plane.)