Selected solutions for worksheets from Math 53 (U.C. Berkeley's multivariable calculus course).

#9. Limits and Continuity

Questions

1.

(a) and (b) are continuous. (c) is not continuous since the function "jumps" at one point. (d) and (e) are continuous everywhere they are defined, but (d) is not defined at one point which has been removed, and (e) is not defined at 0 where the function goes off to infinity.

2.

(a) True. (b) True. (c) False (the denominator may be zero.) (d) True.

Problems

1.

Suppose we're given an $\epsilon > 0$. We want to show that f is continuous at any $\boldsymbol{x} \in \mathbb{R}^3$. Pick any $\boldsymbol{y} \in \mathbb{R}^3$, meaning we can take δ to be as large as we like. Then $|f(\boldsymbol{x}) - f(\boldsymbol{y})| = 0 < \epsilon$. So f is continuous at $\boldsymbol{x}$.

2.

(a) The limit is 1.

(b) The limit is zero.

(c) The limit does not exist. If it did, the limit should be the same regardless of how we approach the origin. However, the answers in (a) and (b) were different.

3.

Note that the function $h(\boldsymbol{x}) = -1$ is continuous since it's constant. Also f is assumed to be continuous. Hence by 2b) we know that $f \cdot h = -f$ is continuous. (Or use that we have a composition of two continuous functions, f and the function sending $\boldsymbol{x} \mapsto -\boldsymbol{x}$.)