

## 8. Functions of Several Variables

### Questions

1.

- (a) All  $(x, y)$  in  $\mathbb{R}^2$  for which  $f(x, y) = c$  and  $g(x, y) = d$ .
- (b) This is empty unless  $c = d$ , in which case the sets are the same.
- (c) The set of all numbers  $z$  for which  $f(x, y) = g(x, y)$ .

3.

Level sets can be pretty weird. For example, the level set of zero for  $f(x, y) = \min\{|x|, |y|\}$  is the union of the  $x$  and  $y$  axes. So at  $(0, 0)$  there is no unique tangent line; how would you pick between the  $x$  and  $y$  axes?

### Problems

1.

You should sketch these, but I am going to give an elucidating description/hint instead.

- (a) All are planes in  $\mathbb{R}^3$ .
- (b) All are ellipsoids in  $\mathbb{R}^3$ .
- (c) All are ellipsoids in  $\mathbb{R}^3$ , and in particular are spheres.
- (d) All are hyperbolas in  $\mathbb{R}^2$ , but not aligned with the  $x$  and  $y$  axes.
- (e) All are hyperbolas in  $\mathbb{R}^2$ .