Selected solutions for worksheets from Math 53 (U.C. Berkeley's multivariable calculus course).

6. Vector Functions and Space Curves

Questions

2.

$\mathbf{r}'(t) = f'(t)\mathbf{i} + g'(t)\mathbf{j}$ is the vector tangent to the curve at t, which means it points in precisely the direction of the line tangent to the curve at t. Slope is "rise over run," so in this case the slope is $\frac{g'(t)}{f'(t)}$.

3.

(a) The graph of $\mathbf{r}(t)$ is a point in space.

(b) The graph of $\mathbf{r}(t)$ is changing at precisely the same speed and in precisely the same direction at all times. Such graphs are lines.

(c) The graph of $\mathbf{r}(t)$ is contained in the surface of a circle or sphere.

(d) The graph of $\mathbf{r}(t)$ is changing direction, but never speed.

Problems

1.

(a) Take the dot product of $\mathbf{r}(t)$ and $\mathbf{r}'(t)$. Since $\mathbf{r}'(t) = -\sin t\mathbf{i} + \cos t\mathbf{j} + 0\mathbf{k}$, this dot product is

$$\mathbf{r}(t) \cdot \mathbf{r}'(t) = (\cos t, \sin t, 0) \cdot (-\sin t, \cos t, 0) = -\cos t \sin t + \sin t \cos t + 0 = 0$$

no matter what t is. So yes, $\mathbf{r}(t) \perp \mathbf{r}'(t)$ for all t.

Now take the dot product of $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$. Since $\mathbf{r}''(t) = -\cos t\mathbf{i} - \sin t\mathbf{j} + 0\mathbf{k}$, this dot product is

$$\mathbf{r}'(t) \cdot \mathbf{r}''(t) = (-\sin t, \cos t, 0) \cdot (-\cos t, -\sin t, 0) = \sin t \cos t - \cos t \sin t + 0 = 0$$

no matter what t is. So yes, $\mathbf{r}'(t) \perp \mathbf{r}''(t)$ for all t.

(b) False. On homework, you showed that the graph of any function for which $\mathbf{r}(t) \perp \mathbf{r}'(t)$ is contained in a sphere. So choose any function whose graph is not contained in a sphere, e.g. $\mathbf{r}(t) = (t^2, 0, 0)$. (Think about why this reasoning works. It is an example of a proof using the *contrapositive*.)

2.

Position: $(10, 100, 1000)$. Velocity: $(1, 20, 300)$. Speed: $\sqrt{90,401}$. Acceleration: $(0, 2, 30)$.

3.

Find some t for which $\mathbf{r}(t) = (1, 1, 1)$, then plug that t into $\mathbf{r}'(t)$. You should get $(1, 1, 1)+ t(2, 3, 4)$ as the tangent line.