Selected solutions for worksheets from Math 53 (U.C. Berkeley's multivariable calculus course).

5. Quadric Surfaces

Questions

1.

The answer is 3, none. To figure this out, think about the following fact: any surface obtained by rotating a curve about an axis will have some circular cross-section (in particular, the intersection of the surface with the plane normal to the axis of rotation will be a circle). There are special cases of all of the surfaces which can be so obtained, but none can in general be obtained this way.

2.

A cylinder.

Problems

1.

All can be solved by completing the square and comparing to the general equations, or by checking the intersection with the planes with x, y, or z constant.

(a) Ellipsoid, because all squared terms are positive and they sum to a positive number.

(b) Elliptic cone, because one squared term is negative and they add to zero.

(c) Hyperboloid of one sheet, because (by adding all x, y, z terms over to the right and constants to the left) you can reduce to having one squared term negative and adding to a positive number.

2.

Use the change of variables $x = u + v$, $y = u - v$ to show that this is a hyperbolic paraboloid.

3.

Show the hint, then show that the surface's intersection with a plane where z is constant and nonzero is a circle (use the change of variables from **Problem 2**). Why does this show that the surface is a cone?