3. Polar Coordinates

Questions

1.

(a) $x = r\cos\theta$, $y = r\sin\theta$.
(b) $r^2 = x^2 + y^2$, $\tan\theta = y/x$.

2.

(a) A line passing through the origin.
(b) A circle centered at the origin.
(c) A spiral starting at the origin, counterclockwise.

Problems

1.

(Multiply the polar equation $r = 2\sin\theta$ by r on both sides.) The Cartesian equation is $x^2 + (y-1)^2 = 1$. The curve is a circle of radius 1 centered at $(0, 1)$.

2.

(First write the cartesian equation $(x-1)^2 + (y-1)^2 = 2$, then substitute in $x = r\cos\theta$, and $y = r\sin\theta$ and simplify by isolating r to one side of the equation.) The polar equation is $r = 2(\cos\theta + \sin\theta)$.

3.

(b) We have $x(\theta) = r(\theta)\cos\theta$, $x(\theta) = r(\theta)\sin\theta$.

$$\text{Slope } = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

Using the given polar equation $r(\theta) = 3 + \cos 4\theta$, we have $\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = -1$.

(c) $\frac{dr}{d\theta} = 0$ at $\theta = n\frac{\pi}{4}$, where $n = ... - 2, -1, 0, 1, 2, ...$
$\frac{dr}{d\theta} = 0$ at a point means that the change in r with respect to the change in θ is 0 (i.e. at this point, when you vary θ by a little bit, r - the distance from the origin - doesn't change). Geometrically, these are all the points on the curve that is tangent to some circle centered at the origin.

4.

(a) No. Arc Length $= \int_{\pi/2}^{\infty} \frac{\sqrt{\theta^2+1}}{\theta^2}\, \mathrm{d}\theta$ does not have a finite value.

(b) Yes. Arc Length $= \int_0^{\infty} \sqrt{2}e^{-\theta}\, \mathrm{d}\theta = \sqrt{2}$.

5.

$$\text{Area } = 4 \cdot \int_0^{\pi/4} \frac{1}{2} a^2 \cos(2\theta)\, \mathrm{d}\theta = a^2$$