Selected solutions for worksheets from Math 53 (U.C. Berkeley's multivariable calculus course).

#20. Surface Area

Problems

1.

Compute f_x and f_y. Then integrate $\sqrt{{f_x}^2 + {f_y}^2 + 1}$ over $0 \le x \le 1$ and $100 - 25x - \sec^2 x \le y \le 100 - \sec^2 x$.

2.

For a picture of this, see page 1016, question 24, in the textbook. Suppose both cylinders have radius a. We see that the planes $y = x$ and $y = -x$ divide the region into four symmetric parts. On the xy-plane, this corresponds to four triangles. To find the surface area, we find the surface area of the graph of the function above one of these triangles, then multiply by 8 (the top and bottom surface in the four symmetric parts.) So our domain of integration is the triangle in the xy-plane with vertices $(0, 0)$, $(a, -a)$ and (a, a). The surface is the graph of $z = \sqrt{a^2 - x^2}$. Define $f(x, y) = \sqrt{a^2 - x^2}$. Then

$$\begin{aligned} f_x &= \frac{-x}{\sqrt{a^2 - x^2}} \\ f_y &= 0 \end{aligned}$$

So using the formula for surface area:

$$\begin{aligned} \int_0^a \int_{-x}^x \sqrt{\frac{x^2}{a^2 - x^2} + 1}\, dy\, dx &= \int_0^a \int_{-x}^x a\sqrt{\frac{1}{a^2 - x^2}}\, dy\, dx \\ &= 2a^2 \end{aligned}$$

Multiplying this by 8 gives the answer of $\boxed{16a^2}$.