Selected solutions for worksheets from Math 53 (U.C. Berkeley's multivariable calculus course).

#19. Applications of Double Integrals

Problems

1.

The density is $\rho = kr$ since the distance of a point (r, θ) in polar coordinates to the center (assuming the center is at the origin) is just r. Then mass is

$$\int_0^{2\pi} \int_0^1 kr^2 \, dr \, d\theta = \frac{2\pi k}{3}$$

2.

First we find the mass m where $\rho = k$ a constant:

$$\int_0^1 \int_{x^2}^{\sqrt{x}} k \, dy \, dx = \frac{k}{3}$$

Then we find the center of mass, where R is the region we're integrating over:

$$\overline{x} = \frac{1}{m} \int \int_R x\rho \, dA = \frac{3}{k} \int_0^1 \int_{x^2}^{\sqrt{x}} xk \, dy \, dx = \frac{9}{20}$$

Similarly we find $\overline{y} = \frac{9}{20}$.