Selected solutions for worksheets from Math 53 (U.C. Berkeley's multivariable calculus course).

17. Double Integrals over General Regions

Questions

1.

The area of D.

Problems

2.

Integrate $\int_0^1 \int_{x^2}^{\sqrt{x}} xy \, dy \, dx$.

3.

(a) $\int_0^a \int_0^{\sqrt{a^2-y^2}} f(x,y) \, dx \, dy$

(b) Since $\frac{4}{9} < \frac{1}{\sqrt{3}}$, the new integral is

$$\int_0^{\frac{4}{9}} \int_{\frac{1}{3}}^{\sqrt{x}} f(x,y) \, dy \, dx + \int_{\frac{4}{9}}^{\frac{1}{\sqrt{3}}} \int_{\frac{1}{3}}^{\frac{2}{3}} f(x,y) \, dy \, dx + \int_{\frac{1}{\sqrt{3}}}^{\frac{\sqrt{2}}{\sqrt{3}}} \int_{x^2}^{\frac{2}{3}} f(x,y) \, dy \, dx$$

Additional Problem

(c) To find the integrand, consider cross-sections parallel to the yz-axis.