Selected solutions for worksheets from Math 53 (U.C. Berkeley's multivariable calculus course).

15. Lagrange Multipliers

Questions

1.

(b) ∇f points one unit to the right, and ∇g points out (away from the origin) at every point. Recall that the vectors $\nabla f(x, y)$ and $\nabla g(x, y)$ ought to start at (x, y).

(c) When y is positive, the ant crawls clockwise; when y is negative, the ant crawls counterclockwise.

(d) $(1, 0)$ maximizes f, and $\nabla f(1, 0)$ and $\nabla g(1, 0)$ point in the same direction. $(-1, 0)$ minimizes f, and $\nabla f(-1, 0)$ and $\nabla g(-1, 0)$ point in opposite directions.

3.

Level sets can be pretty weird. For example, the level set of zero for $f(x, y) = \min\{|x|, |y|\}$ is the union of the x and y axes. So at $(0, 0)$ there is no unique tangent line; how would you pick between the x and y axes?

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f would be constant on the level sets of g.

Problems

1.

$f(x, y) = y$, $g(x, y) = x - 2x^2y - y^2$, $g(x, y) = 0$. Using Lagrange multipliers, we get $y = \frac{1}{2}$ as the maximum value.

2.

Think about (x, y, z) being the vertex of a box centered at the origin (consider only (x, y, z) in the first quadrant). Then the volume of such a box is given by $f(x, y, z) = (2x)(2y)(2z) = 8xyz$, and $g(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Using Lagrange multipliers, we get $(x, y, z) = \left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$ giving a maximum volume of $\frac{8abc}{3\sqrt{3}}$. (Setting any of $x, y, z = 0$ will also give a solution to the Lagrange multipliers equation, but will not maximize volume.)

3.

$f(x, y, z) = A^2 = s(s-x)(s-y)(s-z)$ (use this instead of A to simplify calculations; it is maximized when A is maximized) and $g(x, y, z) = x+y+z$. We are trying to analyze the situation for $g = p$, any number, and the goal is to show that $x = y = z = \frac{p}{3}$ maximizes area.