14. Maximum and Minimum Values

Questions

1.

No. Consider the function $f(x, y) = 3x^2 + 8xy + 3y^2$. Calculation shows that

$$\begin{aligned} \nabla f &= \langle 6x + 8y, 6y + 8x \rangle \\ f_{xx} &= 6 \\ f_{yy} &= 6 \\ f_{xy} &= 8 \end{aligned}$$

Since $\nabla f(0, 0) = \langle 0, 0 \rangle$, $(0, 0)$ is a critical point. Since $f_{xx} = f_{yy} > 0$, f is concave up in both the $x = 0$ and $y = 0$ plane. However, the second derivative test $D = f_{xx} f_{yy} - f_{xy}^2 = -28 < 0$ imply that $(0, 0)$ is a saddle point and not a minimum.

Problems

1.

(a)

$$\begin{aligned} \nabla f &= \langle 5x^4 - 5, 4y^3 - 32 \rangle \\ f_{xx} &= 20x^3 \\ f_{yy} &= 12y^2 \\ f_{xy} &= 0 \end{aligned}$$

Critical points at

$$\begin{aligned} &(1, 2) \;\text{, minimum} \\ &(-1, 2) \;\text{, saddle point} \end{aligned}$$

(b)

$$\begin{aligned} \nabla f &= \langle 3x^2 + 3y, 3y^2 + 3x \rangle \\ f_{xx} &= 6x \\ f_{yy} &= 6y \\ f_{xy} &= 3 \end{aligned}$$

Critical points at

$$\begin{aligned} &(0, 0) \;\text{, saddle point} \\ &(-1, -1) \;\text{, maximum} \end{aligned}$$

(c)

$$\begin{aligned}
\nabla f &= \langle 2xy + 3y - 6x - 4, x^2 + 3x + 2 \rangle \\
f_{xx} &= 2y - 6 \\
f_{yy} &= 0 \\
f_{xy} &= 2x + 3
\end{aligned}$$

Critical points at

$$\begin{aligned}
&(-2, 8) \ \text{, saddle point} \\
&(-1, -1) \ \text{, saddle point}
\end{aligned}$$