*Selected solutions for worksheets from Math 53 (U.C. Berkeley's multivariable calculus course).*

# #11. Tangent Planes and Differentials

## Questions

**1.**

(a) The analogue is that the tangent plane should be horizontal (whereas for functions of one variable the tangent line is horizontal.) The partial derivatives give the slopes of two vectors in the tangent plane, so both partial derivatives should be zero.

(b) $\partial f/\partial x = 2x - 2 = 0$ and $\partial f/\partial y = 2y - 6 = 0$ imply that $(x, y) = (1, 3)$. So this point is a local extremum.

(c) Completing the square gives $(x-1)^2 + (y-3)^2 + 4 = f(x, y)$. This is at a minimum when $x = 1$ and $y = 3$, so we have a local minimum there. The quadric is an upward facing paraboloid.

## Problems

**1.**

The tangent plane for $z = f(x, y) = e^{x-y}$ is obtained from the partial derivatives $f_x = e^{x-y}$ and $f_y = -e^{x-y}$. It is given by

$$\begin{aligned} z - 1 &= f_x(1,1)(x-1) + f_y(1,1)(y-1) \\ \implies z - 1 &= (x-1) - (y-1) = x - y \end{aligned}$$

This intersects the $z$-axis when $x = y = 0$ so $z = 1$. Thus the answer is $(0, 0, 1)$.

**2.**

(a) 50 miles.

(b)$\Delta f \approx f_x(30, 40)\Delta x + f_y(30, 40)\Delta y$, where $f(x, y)$ is the hypotenuse of the triangle: $f(x, y) = \sqrt{x^2 + y^2}$. Here $\Delta x, \Delta y$ are the possible errors of 0.1.

$$\begin{aligned} f_x &= \frac{x}{\sqrt{x^2+y^2}} \\ f_y &= \frac{y}{\sqrt{x^2+y^2}} \end{aligned}$$

Thus $\Delta f \approx \frac{3}{5}(0.1) + \frac{4}{5}(0.1) = 0.14$.

(c) At worst, the measurements of 30 and 40 miles are off by 0.1 miles. So at worst the actual distance is $\sqrt{30.1^2 + 40.1^2}$ or $\sqrt{29.9^2 + 39.9^2}$. The former deviates more from 50 than the latter. It gives the maximal possible error in $f$ of about 0.14 miles.

(d) No. Errors could cancel, for example.