Selected solutions for worksheets from Math 53 (U.C. Berkeley's multivariable calculus course).

#10. Partial Derivatives

Questions

1.

(a) $f_x(1,2) = -2$, $f_y(1,2) = -4$.

(b) This is a downward facing paraboloid, with circular cross sections. To sketch the slopes, think of slicing the graph at the planes $y = 2$ and $x = 1$.

(c) The level curve is a circle. To predict steepness we can see how close or far apart the level curves moving in the x and y direction.

2.

(a) f_x exists everywhere except the y-axis but including the origin.
(b) f_y exists everywhere except the x-axis but including the origin.
(c) f is continuous everywhere except the x and y axes.

3.

(a) These are yz, xz, xy respectively.

(b) We have a box of dimensions x, y, z. Imagine fixing the xy face, and allowing z to increase 1 unit per unit time. Thus in one unit of time the volume of the box has increased an amount xy, i.e. the change in V with respect to z is xy. Similarly for the other two partial derivatives.

Problems

1.

(a) Upward facing cone.
(b) If we take a plane at $x =$ constant or $y =$ constant, where the constant is not zero, then it intersects the cone in a curve which has a unique tangent line, with slope corresponding to the partial derivative. At $x = 0$ or $y = 0$ though, we don't have a tangent line.
(c) $\partial f/\partial x = x/\sqrt{x^2+y^2}$ is continuous everywhere except at the origin, as expected. Similarly for $\partial f/\partial y$.

2.

(a)

$$\partial f/\partial x = \frac{x^4y + 4x^2y^3 - y^5}{(x^2+y^2)^2}$$
$$\partial f/\partial y = \frac{x^5 - 4x^3y^2 - xy^4}{(x^2+y^2)^2}$$

(b) Both zero. For example

$$\lim_{x \to 0} \frac{1}{x}(f(x,0) - f(0,0)) = \lim_{x \to 0} \left(\frac{xy(x^2 - y^2)}{x(x^2 + y^2)} \right)_{y=0} = 0$$

(c) Yes. Converting into polars gives $\partial f/\partial x = r(\cos^4\theta \sin\theta + 4\cos^2\theta \sin^3\theta - \sin^5\theta)$, since the r's cancel. The expression in parentheses is bounded, so as r tends to zero, the whole expression tends to zero. Thus $\lim_{(x,y)\to(0,0)} \partial f/\partial x = \frac{\partial f}{\partial x}(0,0) = 0$. Similarly for $\partial f/\partial y$. And away from the origin we already have continuity from the expressions for the partial derivatives above. Thus the partial derivatives are continuous everywhere.