1.7: Random Derivative Knowledge

One can think of the derivative $f'(a)$ as the **rate of change** of $f(x)$ at $x = a$. Alternately, the change in $f(x)$ per one unit change in x at $x = a$ is given by $f'(a)$.

The **marginal cost** of producing a units of a given item is the cost of producing the $a+1$st item. The derivative $C'(x)$ of a cost function $C(x)$ is a function for the marginal cost. One can similarly talk about **marginal revenue** and **marginal profit**.

Problems

(1) Compute:
- (a) $\frac{d}{ds}f(s,t)$ where $f(s,t) = s^3 + 3st + t^{-5}$.
- (b) y'' where $y = x^5 + 4(x+3)^2$.
- (c) $\frac{d^2}{dx^2}xy + 2xy^4 + 9(2x+2)^3|_{x=-1}$.

(2) A fundraiser for the Berkeley Math Department is bringing in $100 - \frac{1}{3}x^2$ dollars on day x from the start of the fundraiser. What is the rate of change in dollars brought in on the 20th day?

(3) A cost function for calculus textbook production is given by $C(x) = 3x^{\frac{3}{4}}$, which is the cost in dollars of producing x textbooks. What is the cost of producing the tenth textbook?

1.8: Derivative as a Rate of Change

The **average rate of change** of $f(x)$ over the interval $a \leq x \leq b$ is given by

$$\frac{f(b) - f(a)}{b - a}$$

The **instantaneous rate of change** of $f(x)$ at $x = a$ is given by $f'(a)$.

If the interval under consideration is small, we can approximate f by

$$f(a+h) - f(a) \approx f'(a) \cdot h$$

We can understand this by setting $h = b - a$ in the limit definition of the derivative.

Problems

(1) Let $s(t) = 3t^2 + .5t$ be the position of an object moving in a straight line. What is the average velocity of the object between $t = 1$ and $t = 3$? What are the velocity and acceleration at $t = 2$?

(2) Let $d(x) = 4x^{\frac{1}{2}}$ be the position of an object moving in a straight line from $d(0) = 0$. Will the object turn around? If so, give two values $a_0 < a_1$ for which $d(a_0) > d(a_1)$.

(3) Let $d(x) = (x-2)^3 + 3x$ be the position of an object moving in a straight line from $d(0) = 0$. Will the object slow down? If so, give two values $a_0 < a_1$ for which the velocity at a_1 is less than the velocity at a_0.