2.1: Describing Graphs of Functions

An **open interval** (a, b) is all numbers between a and b but not including a and b.

A **closed interval** $[a, b]$ is all numbers between a and b, including a and b.

(1) Define the following terms: check with your group, with Morgan, or with your book to make sure that you are correct.
   (a) increasing and decreasing in an interval; increasing and decreasing at a point
   (b) relative extremum; relative maximum and minimum
   (c) absolute maximum and minimum value
   (d) concave up; concave down (in terms of tangent lines and in terms of change in slope)
   (e) inflection point (in terms of tangent lines and in terms of change in concavity)

(2) Draw the graph of a function defined on $(0, 1)$ with no absolute maximum and no absolute minimum.

(3) Draw the graph of a function $f(x)$ defined on $(1, 2)$ where f has a negative derivative everywhere $f'(x)$ is defined, but is not decreasing.

(4) Draw the graph of a function with an inflection point and a relative extremum at the same point. What do you notice about the derivative of this function at this inflection point/relative extremum?

2.2: First and Second Derivative Rules

In this section, all functions should have continuous first and second derivatives. In particular this means that the functions must be continuous.

(1) If $f'(a) > 0$, must f be increasing at $x = a$? If $f'(a) < 0$, must f be decreasing at $x = a$?

(2) Draw the graph of a function for which there is some a where $f'(a) = 0$ yet f does not have a relative extremum at $x = a$.

(3) If $f''(a) > 0$, must f be concave up at $x = a$? If $f''(a) < 0$, must f be concave down at $x = a$?

(4) Draw the graph of a function for which there is some a where $f''(a) = 0$ yet f does not have an inflection point at $x = a$.

(5) Let $f(x) = \frac{1}{3}x^3 - x$. It may help to draw this graph, identifying the x-intercepts and relative extrema.
   (a) Does f have any inflection points? If so, identify them; if not, explain why not.
   (b) Does f have a maximum value for $-2 < x < 2$? Does f have a minimum value in the same interval? If so, what are they? If not, why not?
   (c) Does f have a maximum value for $-3 < x < 3$? Does f have a minimum value in the same interval? If so, what are they? If not, why not?
   (d) Does f have a maximum value for $x < -2$? Why or why not?