Quiz #7 Solutions

Math 55 with Professor Stankova
Discussion Section #102 with GSI James Moody

Wednesday, the 12th of October 2016
Write your name at the top!

Question 1 [12 points] How many strings are there of four lowercase letters that have the letter y in them? INFO: There are 26 letters in the alphabet.

Make sure to show your work.

Solution 1:
There are 26^4 strings of four letters (26 independent options for the 1st letter, 2nd letter, 3rd letter, and 4th letter).
There are 25^4 strings of four letters which don't have y (25 independent options for each of the four letters).
Thus (by subtracting the complement), there are $\mathbf{26^4 - 25^4}$ strings of four lowercase letters without y.

Solution 2: Break up into four cases: there must be exactly 1, 2, 3 or 4 occurrences of y in the string (and these are mutually exclusive).
If there is exactly 1 occurrence of y, there are four choices for where to put the y, and 25 possibilities for each of the other three letters.
If there are exactly 2 occurrences of y, there are six choices for where to put the two ys, and 25 possibilities for each of the other two letters.
If there are exactly 3 occurrences of y, there are four choices for where to put the three ys, and 25 possibilities for the remaining letter.
If there are exactly 4 occurrences of y, there is only one choice for where to put the four ys, and no remaining choices to make.

Since these possibilities are mutually exclusive and exhaustive, we add up the results to get the final answer:

$$4 * 25^3 + 6 * 25^2 + 4 * 25^1 + 1$$

Question 2 [± 1 **point**] **Each of four days,** you have lunch. Each day for lunch, you choose an entre, a side, and a drink. For the entre, you have a choice of tofu, seitan, or tempeh. For the side, you have a choice of Donald, Hillary, or Giant Meteor. For the drink, you have a choice of orange, apple, or cranberry juice. **How many possible four-day lunch plans are there?**

$12 \cdot 3$ —— $12 + 3$ —— $\mathbf{3^{12}}$ —— 12^3

Question 3 [± 1 **point**] The number of ways to choose a team of 2 people out of 10 possible players is 10! [Factorial, not an exclamation point]

True —or— **False**?

Question 4 [± 1 **point**] For finite sets A and B, $|A \cup B| = |A| + |B| - |A \cap B|$.

True —or— False?