Quiz #5 Solutions

Math 55 with Professor Stankova
Discussion Section #108 with GSI James Moody

Wednesday, the 28th of September 2016
Write your name at the top!

Question 1 [12 points] Find the greatest common divisor of 220, 66, and 60.

The most important thing to notice is that when finding the gcd of a set of numbers numbers, we can subtract any integer multiple of one of the numbers in the set from any other of the number in the set without changing the gcd.

Dividing 220 by 60, we obtain the equation $220 = 3 \cdot 60 + 40$, which gives us that $220 - 3 \cdot 60 = 40$. Thus:

$$gcd(220, 66, 60) = gcd(40, 66, 60)$$

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Dividing 60 by 40, we obtain $60 = 1 \cdot 40 + 20$, which gives us that $60 - 1 \cdot 40 = 20$. Thus:

$$gcd(40, 66, 60) = gcd(40, 66, 20)$$

Dividing 66 by 20, we obtain a remainder of 6, thus:

$$gcd(40, 66, 20) = gcd(40, 6, 20)$$

Dividing 20 by 6, we obtain a remainder of 2, thus:

$$gcd(40, 6, 20) = gcd(40, 6, 2)$$

Dividing 40 by 2, we obtain a remainder of 0, thus:

$$gcd(40, 6, 2) = gcd(0, 6, 2)$$

Dividing 6 by 2 we obtain a remainder of 0, thus:

$$gcd(0, 6, 2) = gcd(0, 0, 2) = 2$$

Thus $gcd(220, 66, 60) = 2$

Question 2 [± 1 **point**] If p is an odd prime, then $\frac{p-1}{2}$ has a multiplicative inverse modulo p.

True —or— False?

Explanation: By Fermat's Little Theorem, any number $a \in \mathbb{N}$ which is not a multiple of p has $a \cdot a^{p-2} \equiv 1 \pmod{p}$, and thus a has an inverse modulo p (namely a^{p-2}). To see that FLT applies here, notice that if $a = \frac{p-1}{2}$, then $0 < a < p$, so a cannot be multiple of p.

Question 3 [± 1 **point**] If p is an odd prime, then 2 has a multiplicative inverse modulo p.

True —or— False?

Explanation: Use FLT (as in problem 3), setting $a = 2$.

Question 4 [± 1 **point**] The integer whose 5-ary expansion is $(10000)_5$ is even.

True —or— **False**?

Explanation: First expand out $(10000)_5 = 0 \cdot 5^0 + 0 \cdot 5^1 + 0 \cdot 5^2 + 0 \cdot 5^3 + 1 \cdot 5^4 = 5^4$. Then notice that an odd number (like five) raised to any power will still be odd (since odd times odd is odd).