# Quiz #2 Solutions

Math 55 with Professor Stankova
Discussion Section #108 with GSI James Moody

Wednesday, the 7th of September 2016
**Write your name at the top!**

**Question 1:** Express the negations of the following statements so that the negation symbol is applied only to predicates (i.e. so that no negation symbol is applied to a complex formula) [4 points each].

(a) $\forall x \exists y (P(x, y) \vee Q(x, y))$

$$\begin{aligned} \neg \forall x \exists y (P(x, y) \vee Q(x, y)) &\equiv \exists x \neg \exists y (P(x, y) \vee Q(x, y)) \\ &\equiv \exists x \forall y \neg (P(x, y) \vee Q(x, y)) \\ &\equiv \exists x \forall y (\neg P(x, y) \wedge \neg Q(x, y)) \end{aligned}$$

(b) $\forall x (P(x) \wedge \exists y Q(y))$

$$\begin{aligned} \neg \forall x (P(x) \wedge \exists y Q(y)) &\equiv \exists x \neg (P(x) \wedge \exists y Q(y)) \\ &\equiv \exists x (\neg P(x) \vee \neg \exists y Q(y)) \\ &\equiv \exists x (\neg P(x) \vee \forall y \neg Q(y)) \end{aligned}$$

(c) $\exists x \exists y (\exists z P(x, y, z) \wedge \forall z \neg Q(x, y, z))$

$$\begin{aligned} \neg \exists x \exists y (\exists z P(x, y, z) \wedge \forall z \neg Q(x, y, z)) &\equiv \forall x \neg \exists y (\exists z P(x, y, z) \wedge \forall z \neg Q(x, y, z)) \\ &\equiv \forall x \forall y \neg (\exists z P(x, y, z) \wedge \forall z \neg Q(x, y, z)) \\ &\equiv \forall x \forall y (\neg \exists z P(x, y, z) \vee \neg \forall z \neg Q(x, y, z)) \\ &\equiv \forall x \forall y (\forall z \neg P(x, y, z) \vee \exists z \neg \neg Q(x, y, z)) \\ &\equiv \forall x \forall y (\forall z \neg P(x, y, z) \vee \exists z Q(x, y, z)) \end{aligned}$$

**Question 2 (T/F):** The negation of the statement “Everyone ate nothing.” is “No one ate everything”.
True — **False**

The negation of “Everyone ate nothing” would be “Someone ate something”

**Question 3 (T/F):** The statement $\exists x(P(x) \wedge Q(x))$ is logically equivalent to $\exists x P(x) \wedge \exists x Q(x)$
True — **False**

Here’s a counter-example where the first formula ends up being false and the second true: Suppose P(x) = “x is big” and Q(x) = “x is small”. Then $\exists x \exists y(P(x) \wedge P(y))$ means “Something is both big and small” (which is false in the real world), while $\exists x P(x) \wedge \exists y Q(x)$ means “Something is big, and something is small” (which is true in the real world).

**Question 4 (T/F):** $\exists x \exists y P(x, y)$ is logically equivalent to $\exists y \exists x P(x, y)$
**True** — False