# Quiz #2 Solutions

Math 55 with Professor Stankova
Discussion Section #102 with GSI James Moody

Wednesday, the 7th of September 2016
**Write your name at the top!**

**Question 1:** Express the negations of the following statements so that the negation symbol is applied only to predicates (i.e. so that no negation symbol is applied to a complex formula) [4 points each].

(a) $\forall x \exists y P(x, y)$

$$\begin{aligned} \neg \forall x \exists y P(x, y) &\equiv \exists x \neg \exists y P(x, y) \\ &\equiv \exists x \forall y \neg P(x, y) \end{aligned}$$

(b) $\exists x(P(x) \rightarrow \exists y Q(y))$

$$\begin{aligned} \neg \exists x(P(x) \rightarrow \exists y Q(y)) &\equiv \forall x \neg(P(x) \rightarrow \exists y Q(y)) \\ &\equiv \forall x(P(x) \wedge \neg \exists y Q(y)) \\ &\equiv \forall x(P(x) \wedge \forall y \neg Q(y)) \end{aligned}$$

(c) $\exists x \forall y(\exists z \neg P(x, y, z) \vee \forall z \neg Q(x, y, z))$

$$\begin{aligned} \neg \exists x \forall y(\exists z \neg P(x, y, z) \vee \forall z \neg Q(x, y, z)) &\equiv \forall x \neg \forall y(\exists z \neg P(x, y, z) \vee \forall z \neg Q(x, y, z)) \\ &\equiv \forall x \exists y \neg(\exists z \neg P(x, y, z) \vee \forall z \neg Q(x, y, z)) \\ &\equiv \forall x \exists y(\neg \exists z \neg P(x, y, z) \wedge \neg \forall z \neg Q(x, y, z)) \\ &\equiv \forall x \exists y(\forall z \neg \neg P(x, y, z) \wedge \exists z \neg \neg Q(x, y, z)) \\ &\equiv \forall x \exists y(\forall z P(x, y, z) \wedge \exists z Q(x, y, z)) \end{aligned}$$

**Question 2 (T/F):** The negation of the statement "Every student has chatted with some other student." is "No student has chatted with another student".

True — **False**

The negation would be "Some student hasn't chatted with any other student"

**Question 3 (T/F):** The statement $\exists x(P(x) \vee Q(x))$ is logically equivalent to $\exists x P(x) \vee \exists x Q(x)$

**True** — False

This is a law, known as "Distribution of Existential Quantification over Disjunction". If you got this wrong, you may want to review which operations distribute over which other operations. You can also prove this particular distributive law as follows:

Proof that $\exists x(P(x) \vee Q(x)) \implies \exists x P(x) \vee \exists x Q(x)$: Suppose $\exists x(P(x) \vee Q(x))$. Then $P(c) \vee Q(c)$ for some c. Breaking up by cases, if P(c) is true (case 1), then $\exists x P(x)$ is true. If Q(c) is true (case 2), then $\exists x Q(x)$ is true. In either case, $\exists x P(x) \vee \exists x Q(x)$ is true.

Proof that $\exists x(P(x) \vee Q(x)) \impliedby \exists x P(x) \vee \exists x Q(x)$: Suppose $\exists x P(x) \vee \exists x Q(x)$. Breaking up by cases, if $\exists x P(x)$ is true (case 1), then $P(c)$ is true for some c, and thus $P(c) \vee Q(c)$ is true for that c. If $\exists x Q(x)$ is true (case 2), then $Q(d)$ is true for some d, and thus $P(d) \vee Q(d)$ is true for that d. In either case $\exists x(P(x) \vee Q(x))$ is true.

**Question 4 (T/F):** $\forall x \exists y P(x, y)$ is logically equivalent to $\exists x \forall y P(x, y)$

True — **False**

To see a counter-example, consider P(x,y) = "person x is in location y". Then $\forall x \exists y P(x, y)$ says "Everyone is located somewhere", while $\exists x \forall y P(x, y)$ says "Someone is located everywhere." These clearly don't mean the same thing!