

Quiz #1 Solutions

Math 55 with Professor Stankova
Discussion Section #102 with GSI James Moody

Wednesday, the 31st of August 2016
Write your name at the top!

Question 1: You travel to an island where you know there live three people: a knight, a knave, and a spy. The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth. You meet each of these three people, but not necessarily in that order. The first person you meet says “I am the knave”. The second person you meet says “I am the knight”. The third person you meet says “ $2 + 2 = 4$ ”. Determine who is the knight, who is the knave, and who is the spy. NOTE: There is a unique solution. (12 points).

SOLUTION 1: There are six possible orders you could have met the three people in:

Order	“I am the knave”	“I am the knight”	“ $2+2=4$ ”	Can we rule it out?
KNIGHT KNAVE SPY	F	F	T	Yes, because the Knight can't lie.
KNIGHT SPY KNAVE	F	F	T	Yes, because the Knight can't lie.
KNAVE KNIGHT SPY	T	T	T	Yes, because the Knave can't tell the truth.
KNAVE SPY KNIGHT	T	F	T	Yes, because the Knave can't tell the truth.
SPY KNIGHT KNAVE	F	T	T	Yes, because the Knave can't tell the truth.
SPY KNAVE KNIGHT	F	F	T	...

The only possible order remaining (after ruling out the others) is Spy, Knave, Knight.

SOLUTION 2: The knight would never say “I am the knave”, because then he would be lying. The knave would never say “I am the knave”, because then he would be telling the truth. Thus (by process of elimination), the first person must be the spy.

The knave would never say “ $2+2=4$ ”, because then he would be telling the truth. The spy can't have said “ $2+2=4$ ”, because we already proved he was the first person we met. Thus the third person must be the knight.

By process of elimination, the second person must be the knave.

Question 2 (T/F): $\neg\neg p$ is equivalent to p

True — False

Question 3 (T/F): $p \rightarrow q$ is equivalent to $q \rightarrow p$

True — **False**

Question 4 (T/F): $p \vee \neg q$ is equivalent to $q \rightarrow p$

True — False

HOW TO SOLVE THESE: Write out the truth tables, and check if the columns for both formulas are the same. If so, the two formulas are equivalent. If not, they aren't.