Quiz #10 Solutions

Math 55 with Professor Stankova
Discussion Section #102 with GSI James Moody

Wednesday, the 2nd of November 2016
Write your name at the top!

Question 1 [12 points] Suppose you flip three coins at once. Are the following two events independent?

E1: There is at least one head and at least one tail.
E2: There is at most one head.

Explain your reasoning and/or draw a picture, or else receive no credit. A simple "Yes/No" answer will not suffice!

First let's just count the sizes of all the sets involved.
The sample space Ω is all length-3 strings of H and T (for heads and tails).
So, $|\Omega| = 2^3 = 8$.
E_1 consists of all the strings in Ω except HHH and TTT. So $|E_1| = 8-2 = 6$.
E_2 consists of the strings TTT, HTT, THT, and TTH So $|E_2| = 4$.
$E_1 \cap E_2$ consists of the strings HTT THT, and TTH, so $|E_1 \cap E_2| = 3$.

Finally we find

$$p(E_1 \cap E_2) = \frac{|E_1 \cap E_2|}{|\Omega|} = \frac{3}{8} = \frac{3}{4} \cdot \frac{1}{2} = \frac{|E_1|}{|\Omega|} \cdot \frac{|E_2}{|\Omega|} = p(E_1) \cdot p(E_2)$$

Thus E_1 and E_2 are independent.

Question 2 [± 1 **point**] If A and B are independent events, then $p(A \cup B) = p(A) + p(B) - p(A) \cdot p(B)$.

True —or— False?

Question 3 [± 1 **point**] If A and B are independent events, then $p(A|B) = p(B|A)$.

True —or— **False**?

Question 4 [± 1 **point**] If A and B are independent events, then $p(A \cap B) = p(A) + p(B)$.

True —or— **False**?