11/1

1) Induction (2) Countiy (3) Probability

1) ① Induction:

If $P(1)$ is true, and whenever $P(n)$ is true, $P(n+1)$ must also be true,
Then $P(n)$ is true for positive integers

② Strong induction

If $P(1)$ is true, and whenever $P(k)$ is true for all $k \le n$, then $P(n+1)$ is true.
Then $P(n)$ is true for all $n \in \mathbb{Z}^+$.

ex1) Use Pascal's identity + Induction, show:

$$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$

P.I $\quad \binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1} \qquad 0 \le k \le n-1$

Base case: $n=1$

$$\sum_{k=0}^{1} \binom{1}{k} = \binom{1}{0} + \binom{1}{1} = 1 + 1 = 2^1 \quad ✓$$

IH: Suppose for some $n$ that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$

Consider

$$\sum_{k=0}^{n+1} \binom{n+1}{k} = 1+1 + \sum_{k=1}^{n} \binom{n+1}{k} = 1+1 + \sum_{k=1}^{n} \left(\binom{n}{k} + \binom{n}{k-1}\right)$$

$$= 1+1+\sum_{k=1}^{n} \binom{n}{k} + \sum_{k=1}^{n} \binom{n}{k-1}$$

$$= \left(1 + \sum_{k=1}^{n} \binom{n}{k}\right) + \left(1 + \sum_{k=1}^{n-1} \binom{n}{k}\right)$$

$$= \sum_{k=0}^{n} \binom{n}{k} + \sum_{k=0}^{n} \binom{n}{k}$$

by IH

$$= 2^n + 2^n = 2^{n+1} \quad ✓ \quad \blacksquare$$

ex2) Prove using strong induction that $\sqrt{2}$ is not rational.
Show it using

$P(n) = $ " $\sqrt{2} \neq \frac{n}{b}$ for all $b \in \mathbb{Z}^+$ "

Base case:
WTS $P(1)$: suppose not. so for some $b \in \mathbb{Z}^+$,
$\sqrt{2} = \frac{1}{b}$. $2 = \frac{1}{b^2}$, $1 = 2b^2$. 1 is even $\lightning$
so $P(1)$ is true

SIH: Suppose for some $k$, $P(n)$ is true for all $n \leq k$,
WTS: $P(k+1)$ is true.

Assume $P(n)$ is true for all $n \leq k$
Towards contradiction, $P(k+1)$ is false.

i.e, $\sqrt{2} = \frac{k+1}{b}$ for some $b \in \mathbb{Z}^+$

$2 = \frac{k^2+2k+1}{b^2}$ $2b^2 = (k+1)^2$, $k+1$ is even, $k+1 = 2l$
$2b^2 = 4l^2$
$b^2 = 2l^2$ $\Rightarrow$ so $b$ is even

so if $\sqrt{2} = \frac{k+1}{b}$, $= \frac{(\frac{k+1}{2})}{(\frac{b}{2})}$ $P(\frac{k+1}{2})$ is false $\lightning$

Contradict the inductive hypothesis that
$P(n)$ is true for $\forall n \leq k$

3) Well-ordering Principle
→ Usually prove by contradiction
1) assume $n$ is the least # s.t $P(n)$ is true
2) Prove that for some $k < n$, $P(k)$ is still true
$\lightning$ w/ well-ordering Principle

2) Counting

~~2-0)~~ $n$ balls, $k$ boxes.

2-1) $n$ distinguishable balls, $k$ distinguishable boxes

$k^n$ permutations (Permutation w/ replacement)
(for each balls, there are $k$ choices)

2-2) $n$ indistinguishable balls into $k$ dist. boxes.

$\binom{n+k-1}{k-1}$. o | o o | o o | o o

( $k-1$ bars) (Combination w/ replacement)

same as $\binom{n+k-1}{n}$.

2-3) $n$ dist. balls into $k$. indist. boxes.

Use the stirling #.

2-4) $n$ indistinguitive into $k$ indist. boxes
partitions of $n$ by $k$
$k_1 + k_2 + \cdots k_k = n$.
$\to$ ways to partition

Ex1. How many ways to choose #s of 8 dogs from an infinite collection of A, B, C.

$\binom{8+3-1}{3-1} = \binom{10}{2} = \binom{10}{8} = 45$ ways.

Ex2) How many words can be made by "MISSOURI"

$\frac{8!}{2!2!}$

Ex3) $n$ distingui. objects into $k$ dis. boxes so that $n_i$ objects go into box $i$

$$\frac{n!}{n_1!\, n_2! \cdots n_k!}$$

Ex4) How many ways are there to deal 4 ppl 5 card hands

$$\frac{52}{5!\,5!\,5!\,5!\,32!} \qquad (5+5+5+5+32)=52$$

Prob.

Ex) ~~Count # of ways~~ of drawing 3 of a kind but not a 4 of a kinds and not a full house

$\binom{13}{3}$ choices for the kinds

3 choices for which one is the triple

$\binom{4}{3}$ choices for the triple, 4 for one single cards 4 of the other.

$$\frac{\binom{13}{3}\, 3 \binom{4}{3} 4^2}{\binom{52}{5}}$$

3) Probability

① Bernoulli trial

Ex) Zvezda puts 13 T/F questions
you have no clue (LOL)
but have a 1/3 chance to guess correctly
what's p(8 are right)

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3) (2) E, F independent

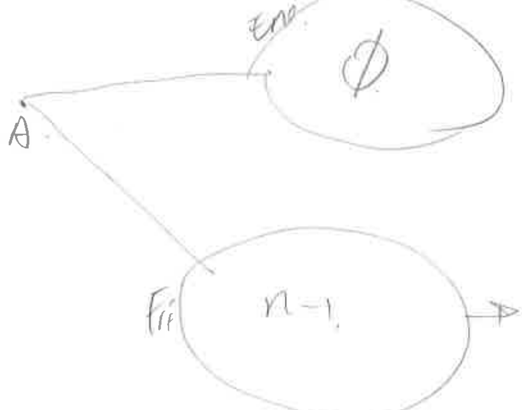
$P(E \wedge F) = P(E) \cdot P(F)$

$$P(E \mid F) = \frac{P(E \wedge F)}{P(F)}$$

(3) Rasmey Number.

$R(m,n)$ is smallest # $k$ s.t.
in $k$ ppl, there are $m$ mutual enemies
or ~~and~~ $n$ mutual friends.

Ex) show if $n \geq 2$. $R(n,2) = n$




→ all friends.