On the distribution of arithmetic sequences in the Collatz graph

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The $3x + 1$ conjecture (Collatz conjecture)

- Famous open problem stated in 1929 by Collatz

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- Define $C(x) = \begin{cases} x/2 & x \text{ is even} \\ 3x+1 & x \text{ is odd} \end{cases}$.

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The $3x + 1$ conjecture (Collatz conjecture)

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- Define $C(x) = \begin{cases} x/2 & x \text{ is even} \\ 3x+1 & x \text{ is odd} \end{cases}$.
- What is the long-term behaviour of C as a discrete dynamical system?

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