

# 3D Geometry and Vectors

Math 53, section 213

September 12, 2014

1. What 3D figure does the equation  $x^2 + z^2 \leq 9$  represent? Sketch it on coordinate axes.
2. Show that the equations  $x^2 + y^2 = z^2$ ,  $z \geq 0$  represents an infinite, hollow cone pointing upwards along the  $z$ -axis.
3. Write down equations that describe an ice cream cone: that is, a filled in cone with a solid hemisphere sitting on top.

4. Find the cosine of the angle between the vectors  $\langle 1, 2, 3 \rangle$  and  $\langle 4, 5, 6 \rangle$  in the coordinate plane.
  
5. Find the angle  $\angle ABC$  where  $A = (1, 0, 0)$ ,  $B = (0, 1, 0)$ , and  $C = (0, 0, 1)$ .
  
6. Compute the cross product of  $\langle 1, 1, -1 \rangle$  and  $\langle 2, 4, 6 \rangle$ . What is the area of the parallelogram spanned by these vectors?
  
7. Use the scalar product  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$  to derive the formula for the volume of a rectangular prism (box) and verify that it is indeed the product of the length, width, and height.
  
8. What is the volume of the parallelepiped spanned by  $\langle 1, 2, 3 \rangle$ ,  $\langle 4, 5, 6 \rangle$ , and  $\langle 1, 3, 6 \rangle$ ?