# Quiz 6

Math 53, section 213

October 29, 2014

1. Use a double integral to find the area of the region of points that lie inside the circle $(x-1)^2 + y^2 = 1$ and outside the circle $x^2 + y^2 = 1$.

   **Solution:** The first circle can be expressed in polar coordinates as $r = 2\cos\theta$, and the second is $r = 1$. The circles intersect at $(1/2, \pm\sqrt{3}/2)$, which are at angles $\pm\pi/3$ from horizontal. Therefore, the integral can be expressed as

$$\begin{aligned}
\int_{-\pi/3}^{\pi/3}\int_{1}^{2\cos\theta} r\,dr\,d\theta &= \int_{-\pi/3}^{\pi/3} \frac{1}{2}r^2 \,\Big|_{1}^{2\cos(\theta)}\, d\theta \\
&= \int_{-\pi/3}^{\pi/3} 2\cos^2(\theta) - \frac{1}{2}\,d\theta \\
&= \int_{-\pi/3}^{\pi/3} \cos(2\theta) + 1 - \frac{1}{2}\,d\theta \\
&= \pi/3 + \frac{1}{2}\sin(2\theta)\,\Big|_{-\pi/3}^{\pi/3} \\
&= \pi/3 + \sqrt{3}/2.
\end{aligned}$$

2. Find the surface area of the part of the plane $3x + 2y + z = 6$ that lies in the first octant.

**Solution:** The plane intersects the first octant in a triangle with vertices $(2, 0, 0)$, $(0, 3, 0)$, and $0, 0, 6$ since these are the intercepts with the positive x, y, and z axes respectively. Thus this is the surface area of the part of the surface $z = 6 - 3x - 2y$ over the region $0 \leq x \leq 2$, $0 \leq y \leq 3 - 3x/2$.

The derivatives of the function $6 - 3x - 2y$ in the $x$ and $y$ directions are $-3$ and 2, and so the surface area is

$$\int_0^2 \int_0^{3-3x/2} \sqrt{1 + (-3)^2 + (-2^2)}\, dy\, dx = 3\sqrt{14}.$$