Quiz 5

Math 53, section 213

October 15, 2014

1. Find the volume of the solid that lies under the plane $4x + 6y - 2z + 15 = 0$ and above the rectangle $R = \{(x, y) \mid -1 \leq x \leq 2, -1 \leq y \leq 1\}$.

   Solution: Solving for z, we find that $z = 2x + 3y + 15/2$ is the function defining the plane. To find the volume under this plane over the region R, first note that it is always positive on this region: the smallest z can be over R is when $x = y = -1$, in which case $z = 2.5 > 0$. Thus the volume is the double integral over the rectangle of the function z of x and y:

$$\begin{aligned}
\int_{-1}^{2}\int_{-1}^{1} 2x + 3y + 15/2\, dy\, dx &= \int_{-1}^{2} (2xy + 3y^2/2 + 15y/2)\ |_{-1}^{1}\ dx \\
&= \int_{-1}^{2} 4x + 15\, dx \\
&= (2x^2 + 15x)\ |_{-1}^{2} \\
&= 2 \cdot (4 - 1) + 15 \cdot (2 - (-1)) \\
&= 51.
\end{aligned}$$

   So, the volume is 51.

2. Evaluate the double integral

$$\iint_D 2xy\, dA$$

where D is the triangular region with vertices $(0,0)$, $(1,2)$, and $(0,3)$.

Solution: The triangle can be realized as the region bounded by the functions $2x$ and $3-x$ on the interval $0 \leq x \leq 1$. So, the integral of $2xy$ over this region is

$$\begin{aligned}
\int_0^1 \int_{2x}^{3-x} 2xy\, dy\, dx &= \int_0^1 xy^2 \,|_{2x}^{3-x}\; dx \\
&= \int_0^1 x(3-x)^2 - x(2x)^2\, dx \\
&= \int_0^1 9x - 6x^2 - 3x^3\, dx \\
&= \frac{9}{2}x^2 - 2x^3 - \frac{3}{4}x^4 \,|_0^1 \\
&= \frac{9}{2} - 2 - 3/4 \\
&= 7/4.
\end{aligned}$$