Quiz 4
Math 53, section 213
October 13, 2014

1. Level curves are shown for a function $f$. Determine whether the following partial derivatives are positive or negative at the point $P$. (2 points each) (See diagram in book, section 14.3 number 74.)

(a) $f_x$
(b) $f_y$
(c) $f_{xx}$
(d) $f_{xy}$
(e) $f_{yy}$

Solution: (a) Negative, since the level curves have decreasing values as one moves to the right from $P$. (b) Positive, since the level curve values increase as one moves directly up from $P$. (c) Positive, since the level curves become farther apart as one moves to the right from $P$, and the slopes are negative by part (a); hence the horizontal cross-section at $P$ is concave up. (d) Positive. The second derivative $f_{xy}$ is the partial derivative with respect to $y$ of the horizontal slopes $f_x$. These slopes start out negative and become less steep as one moves directly upwards from $p$, since the level curves become further away from one another. It follows that the change in the slopes is positive, and so $f_{xy}$ is positive. (e) Positive, since the level curves become closer together as one moves upwards from $P$, and hence the change in slope along the $y$ direction is positive.
2. If $z = f(x, y)$ where $f$ is differentiable, and $x = g(t)$, $y = h(t)$, $g(3) = 2$, $h(3) = 7$, $g'(3) = 5$, $h'(3) = -4$, $f_x(2, 7) = 6$, and $f_y(2, 7) = -8$, find $dz/dt$ when $t = 3$.

Solution: Using the Chain Rule, we find

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t} = f_x(2, 7)g'(3) + f_y(2, 7)h'(3).$$

This evaluates to

$$6 \cdot 5 + (-8) \cdot (-4) = 62.$$