Quiz 4

Math 53, section 213

October 13, 2014

1. Level curves are shown for a function f. Determine whether the following partial derivatives are positive or negative at the point P. (2 points each) (See diagram in book, section 14.3 number 74.)

   (a) f_x

   (b) f_y

   (c) f_{xx}

   (d) f_{xy}

   (e) f_{yy}

   Solution: (a) Negative, since the level curves have decreasing values as one moves to the right from P. (b) Positive, since the level curve values increase as one moves directly up from P. (c) Positive, since the level curves become farther apart as one moves to the right from P, and the slopes are negative by part (a); hence the horizontal cross-section at P is concave up. (d) Positive. The second derivative f_{xy} is the partial derivative with respect to y of the horizontal slopes f_x. These slopes start out negative and become less steep as one moves directly upwards from p, since the level curves become further away from one another. It follows that the change in the slopes is positive, and so f_{xy} is positive. (e) Positive, since the level curves become closer together as one moves upwards from P, and hence the change in slope along the y direction is positive.

2. If $z = f(x, y)$ where f is differentiable, and $x = g(t)$, $y = h(t)$, $g(3) = 2$, $h(3) = 7$, $g'(3) = 5$, $h'(3) = -4$, $f_x(2, 7) = 6$, and $f_y(2, 7) = -8$, find dz/dt when $t = 3$.

   Solution: Using the Chain Rule, we find

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = f_x(2, 7)g'(3) + f_y(2, 7)h'(3).$$

   This evaluates to

$$6 \cdot 5 + (-8) \cdot (-4) = 62.$$