Quiz 3

Math 53, section 213

September 22, 2014

1. Find parametric equations for the tangent line to the curve given by parametric equations

$$x = 1 + 2\sqrt{t}, y = t^3 - t, z = t^3 + t$$

at the point $(3, 0, 2)$.

Solution: To find the direction of the tangent line, we can take the derivative of the vector function with respect to t, to obtain

$$\langle \frac{1}{\sqrt{t}}, 3t^2 - 1, 3t^2 + 1 \rangle$$

for the derivative at time t. The point $(3, 0, 2)$ lies on the curve exactly at time $t = 1$ (and no other times), and so the tangent vector at this point is $\langle 1, 2, 4 \rangle$. Therefore, a vector equation for the tangent line is given by $\langle 3, 0, 2 \rangle + s \cdot \langle 1, 2, 4 \rangle$. Turning this into a parametric equation we have

$$x = 3 + s, y = 2s, z = 2 + 4s.$$

2. Find the limit, if it exists, or show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}.$$

Solution: Fixing $x = 0$ and taking the limit as $y \to 0$ we find that the limit evaluates to $-4/2 = -2$. But fixing $y = 0$ and taking the limit as $x \to 0$ we find that the limit is 0. Since the limit is different along two different paths towards $(0, 0)$, the limit does not exist.