Quiz 1

Math 53, section 213

September 10, 2014

1. For the parametric curve given by $x = t^2 + 1$, $y = t^2 + t$, find dy/dx and d^2y/dx^2 (when they exist) in terms of t. For which values of t is the curve concave upward?

   Solution: We have $\frac{dx}{dt} = 2t$ and $\frac{dy}{dt} = 2t + 1$, so

   $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t+1}{2t} = \boxed{1 + \tfrac{1}{2}t^{-1}.}$$

   Then, using the fact that $\frac{d^2y}{dx^2}$ is the derivative $\frac{d}{dx}\frac{dy}{dx}$, we obtain

   $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(dy/dx)}{dx/dt} = \frac{-t^{-2}/2}{2t} = \boxed{-t^{-3}/4.}$$

   The curve is concave upward precisely when the second derivative is positive, i.e. when

   $$-t^{-3}/4 > 0.$$

   The left hand side is positive precisely when $\boxed{t < 0}$.

2. Identify the curve

$$r^2 \cos(2\theta) = 1$$

by finding a Cartesian equation for the curve.

Solution: We can first expand $\cos(2\theta)$ as $\cos^2(\theta) - \sin^2(\theta)$ to rewrite the equation as

$$r^2 \cos^2(\theta) - r^2 \sin^2(\theta) = 1.$$

Then since $x = r\cos(\theta)$ and $y = r\sin(\theta)$ we have $x^2 = r^2 \cos^2(\theta)$ and $y^2 = r^2 \sin^2(\theta)$. Substituting into the equation above, we obtain the equation $x^2 - y^2 = 1$, which is a hyperbola.