Midterm 1 Review Worksheet

Math 1A, section 103

February 13, 2014

0. (Warmup.) What is $\lim_{x\to 1} x$?
1. Can you prove that $\lim_{x\to 1} x = 1$ using the ϵ, δ definition of a limit?
2. Find the domain of the function $\sqrt{x} + \log(3 - x)$.
3. Find a formula for the inverse of the function $f(x) = 2^{3^x}$. What is the domain of f^{-1}?
4. Evaluate the limit
$$\lim_{x\to 5} \frac{x^2 - 25}{x - 5}$$
5. Evaluate the limit
$$\lim_{x\to 5} \frac{2^x}{x - 5}.$$
6. Show that the equation $\cos(x) = x$ has at least one positive real solution.
7. Show that $\lim_{x\to 0^+} \log_{10}(2x) = -\infty$ using the definition of a limit.
8. We know that $\lim_{x\to 10} \log_{10}(x) = 1$. Given $\epsilon > 0$ with $\epsilon < 1$, find δ so that whenever $|x - 10| < \delta$, we can guarantee that $|\log_{10}(x) - 1| < \epsilon$.
9. Valentine's Day Bonus: Is there a function whose graph looks like a heart? Why or why not? Can you find an explicit *equation* whose graph looks like a heart? (For instance, the graph of $x^2 + y^2 = 1$ is a circle, which is getting close.)