Substitution in Integrals

Math 1A, section 103

April 24, 2014

0. (Warmup.) Find the integral

$$\int x^2\,dx.$$

1. Compute

$$\int \tan^2(x)\sec^2(x)\,dx$$

2. Evaluate the definite integral

$$\int_0^1 \frac{e^{5x}}{1+e^{5x}}\,dx.$$

3. (a) Evaluate

$$\int \sin^3(x)\,dx$$

by writing $\sin^2(x) = 1 - \cos^2(x)$ and making a substitution.

(b) Use a similar method to evaluate

$$\int \sin^5(x)\,dx.$$

(c) What goes wrong when you try to use this method on

$$\int \sin^2(x)\,dx?$$

(d) Recall the trig identities $\sin^2(x) = \frac{1-\cos(2x)}{2}$ and $\cos^2(x) = \frac{1+\cos(2x)}{2}$. Use these to compute

$$\int \sin^2(x)\,dx.$$

4. (Challenge problem.) Pretend it is 5000 BC and no humans yet know the area of a circle. How could you figure out the area of a circle of radius 1 using integrals? Try to evaluate the integral using "reverse u-substitution", that is, express your integral using the variable u, and make the substitution $u = \sin(x)$ to express it as an integral over x.