

Functions worksheet

Math 1A, section 106

February 4, 2014

0. (Warmup.) If $f(x) = 2x$, what is $f(2)$?
1. How is the graph of $y = 2\sin x + 1$ related to the graph of $y = \sin x$? Sketch the graph of the former.
2. How is the graph of $y = \sqrt{x}$ related to the graph of $y = \sqrt{x-1}$? Sketch the graph of the former and find its domain.
3. If $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$, find a function f such that $f \circ g = h$.
4. If $f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$, find a function g such that $f \circ g = h$.
5. Find the inverse of the function $f(x) = 3x + 5$. What is $f^{-1} \circ h$ where $h(x) = 3x^2 + 3x + 2$ as above? How does this relate to the answer to the previous problem?
6. Find the inverse of the function $f(x) = 2 \cdot 3^{x/4}$. What is the domain of the inverse function?
7. Find the inverse of the function $f(x) = 2x^2$, which is defined on the domain $x \leq 0$.
8. What is the domain of the function $f(x) = \log_2(\log_3(x))$? What is its range? Is it one-to-one? If so, what is its inverse?
9. Does the function $f(x) = x^4 + 2x^2 + 1$, defined on \mathbb{R} , have an inverse? If so, what is its inverse?

10. Find the inverse f^{-1} of the function $f(x) = 3x + 5$, and compute $f^{-1} \circ f$ and $f \circ f^{-1}$. What do you get in either case? Is there a general principle here?
11. Johnny has a play-doh collection, and each year he decides to use his allowance to buy half as much play-doh as he currently has and add to his collection. If he started with three pounds of play-doh when he was 5 years old, approximately how much play-doh will he have when he is 10 years old, to the nearest pound? (You may use a calculator to find an approximate answer.)
12. In music, why do we have a twelve tone scale?
 - (a) It is known that simple ratios of frequencies sound nice together to our ears. For instance, when we move up an octave in pitch from the middle C on a piano to the C above, the sound wave doubles in frequency. To move up to the G above middle C, the frequency is multiplied by 1.5. If the frequency of middle C is 261.6, what is the frequency of the C above that? Of the G just above it?
 - (b) On a piano, there are seven half-steps from the C to the G above it. Use a calculator to compute $2^{7/12}$. What is it close to?
 - (c) Given this, what do you think happens to the frequency when we move up a half step on a piano?
13. A confused cell with a terrifying genetic mutation divides into two cells exactly one minute after being created. Then, after another minute each of the two new cells will divide, and so on. Furthermore, at any moment in time that some cells undergo a division, three extra cells with the same terrifying genetic mutation are brought forth from the Void. So, at time $t = 0$ there is 1 cell, at $t = 1$ minute there are $2 + 3 = 5$ cells, and at time $t = 2$ minutes there are $2 \cdot 5 + 3 = 13$ cells. Find a formula for the number of cells as a function of time in minutes.