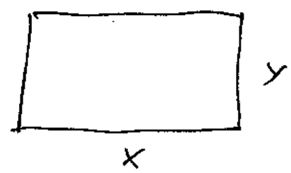
Quiz 9

Math 1A, section 103

April 3, 2014

1. (a) Show that of all the rectangles with a given area, the one with smallest perimeter is a square.
   (b) Show that of all the rectangles with a given perimeter, the one with greatest area is a square.

a) Let A be a fixed positive area.
If a rectangle with width x and height y has area A, we have $A = xy$. The perimeter is $P = 2x + 2y$, and since $y = \frac{A}{x}$ we can write $P = 2x + \frac{2A}{x}$. This is minimized when $\frac{dP}{dx} = 0$, i.e.



$$2 + \frac{(-2A)}{x^2} = 0$$

$$2x^2 = 2A$$

$$x = \sqrt{A}.$$

When $x = \sqrt{A}$, $y = \frac{A}{x} = \frac{A}{\sqrt{A}} = \sqrt{A}$, so it is a square.

b) If P is now fixed and A varies, we have $A = x\left(\frac{P-2x}{2}\right)$, which is maximized when $\frac{dA}{dx} = 0$:

$$\frac{d}{dx}\left(\frac{Px - 2x^2}{2}\right) = \frac{P-4x}{2} = 0 \Rightarrow x = \frac{P}{4}.$$

When $x = P/4$, $y = \frac{P-2x}{2} = P/4$ as well, so again $x = y$ and it is a square.