

Quiz 5

Math 1A, section 103

February 20, 2014

1. (a) Use the Quotient Rule to differentiate the function

$$f(x) = \frac{\tan(x) - 1}{\sec x}$$

- (b) Simplify the expression for $f(x)$ by writing it in terms of $\sin(x)$ and $\cos(x)$, and then find $f'(x)$.

- (c) Show that your answers to parts (a) and (b) are equivalent.

$$\begin{aligned} \text{a) } f'(x) &= \left(\frac{\tan x - 1}{\sec x} \right)' = \frac{\sec x \cdot (\tan x - 1)' - (\tan x - 1)(\sec x)'}{\sec^2 x} \\ &= \frac{\sec x (\sec^2 x + 0) - (\tan x - 1)(\sec x \cdot \tan x)}{\sec^2 x} \\ &= \frac{\cancel{\sec x} ((\sec x)^2 - (\tan x)^2 + \tan x)}{\cancel{\sec x} \cdot \sec x} = \boxed{\frac{1 + \tan x}{\sec x}} \end{aligned}$$

$$\begin{aligned} \text{b) } f(x) &= \frac{\frac{\sin x}{\cos x} - 1}{\frac{1}{\cos x}} = \sin x - \cos x \\ &\Rightarrow f'(x) = \boxed{\cos x + \sin x} \end{aligned}$$

- c) The answer to (a) simplifies as

$$\frac{1 + \frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \boxed{\cos x + \sin x}, \text{ which matches part (b).}$$