

Quiz 3

Math 1A, section 106

February 13, 2014

1. Prove that

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = 5$$

using the ϵ, δ definition of a limit.

Let $\epsilon > 0$. Let $\delta = \epsilon$.

Suppose x is a number satisfying
 $0 < |x - 2| < \delta$.

Then $0 < |x^2 + x - 6 - 5| < \delta$.

Since $|x - 2| > 0$, $x \neq 2$, so we can write this as

$$0 < \left| \frac{(x-2)(x+3)}{(x-2)} - 5 \right| < \delta$$

$$\Rightarrow 0 < \left| \frac{x^2 + x - 6}{x - 2} - 5 \right| < \delta$$

Since $\delta = \epsilon$, by substitution we get

$$\left| \frac{x^2 + x - 6}{x - 2} - 5 \right| < \epsilon.$$

Therefore, by the definition of limit,
$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = 5.$$