

Quiz 12

Math 1A, section 103

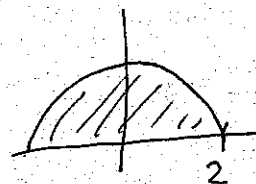
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1. Evaluate $\int_{-2}^2 (x+3)\sqrt{4-x^2} dx$ by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

$$\int_{-2}^2 (x+3)\sqrt{4-x^2} dx = \int_{-2}^2 x\sqrt{4-x^2} dx + 3\int_{-2}^2 \sqrt{4-x^2} dx$$

The integral $\int_{-2}^2 \sqrt{4-x^2} dx$ is the area under a semicircle of radius 2, which is 2π . So

$$3\int_{-2}^2 \sqrt{4-x^2} dx = 3 \cdot 2\pi = 6\pi.$$



To evaluate $\int_{-2}^2 x\sqrt{4-x^2} dx$, make the substitution $u = 4-x^2$, so $du = -2x dx$. Then $dx = -\frac{1}{2x} du$, and the integral becomes

$$\int_0^0 -\frac{1}{2} \sqrt{u} du = 0$$

So the total integral is $0 + 6\pi = \boxed{6\pi}$.