Derivatives Practice Worksheet

Math 1A, section 103

February 27, 2014

0. (Warmup.) Find the slope of the tangent line at the point $(2, 4)$ on the curve $y = x^2$.

1. Let $f(x) = x^3 - 2x$.

   (a) Draw the graph of $f(x)$. Where, precisely, does it cross the x-axis?

   (b) Using your graph from part (a), sketch the graph of the derivative $f'(x)$.

   (c) Now take the derivative of f using derivative rules, and draw the resulting graph. Does it match your picture from part (b)?

2. Compute the derivative of the following functions with respect to x.

   (a) $f(x) = \frac{\sin(x)}{1+\cos(x)}$

(b) $f(x) = \tan(x)^2$

(c) $f(x) = \frac{6x^3+3x-1}{x^3}$

(d) $f(x) = \frac{\sin(x)}{x}$

(e) $f(x) = \sin(x)\cos(y)$

3. **Chain rule**: The chain rule says that the derivative of the composition of two functions, $f(g(x))'$, is equal to $g'(x) \cdot f'(g(x))$. Use the chain rule to compute the derivatives of the following functions:

   (a) $f(x) = \sin(2x)$

   (b) $f(x) = \tan(x)^2$

   (c) $f(x) = e^{3x^2}$

   (d) $f(x) = 1/(1+x)$

   (e) $f(x) = \sin(\cos(x))$

4. Derive the "triple chain rule": what is the derivative of $f(g(h(x)))$?

5. An iPhone is thrown into the air, starting from a height of 1 meter off the ground, and with an initial velocity of 2 meters per second (written $2m/s$). Gravity slows it down by an acceleration of 9.8 meters per second squared ($9.8m/s^2$).

   (a) What is the maximum height reached by the iPhone, and how long does it take the iPhone to reach that height? (Hint: The object's height as a function of time is given by $h(t) = h_0 + vt - \frac{1}{2}at^2$ where h_0 is the initial height, v is the initial velocity, and a is the velocity.)

   (b) If you don't catch the iPhone on the way down, how long does it take for it to smash against the ground?