Derivatives Worksheet 1

Math 1A, section 103

February 20, 2014

0. (Warmup.) If $f(x) = 1$, what is $f'(2)$?

1. Compute the derivatives of the following functions using the limit definition of derivative.

   (a) $f(x) = 2x + 3$
   (b) $f(x) = x^2$
   (c) $f(x) = x^3 - x + 1$
   (d) $f(x) = \sqrt{1 - x}$
   (e) $f(x) = x^{3/2}$
   (f) $f(x) = \frac{1}{x^2}$
   (g) $f(x) = x + \frac{1}{x}$

2. Describe two ways in which a derivative can fail to exist.

3. Where is the function $f(x) = |x - 6|$ not differentiable?

4. In problem 1, you computed the derivative of $f(x) = x^2$. Check your answer by plotting the graph of $f(x)$ and estimating the slope of the tangent line at $x = 1$.

5. **Discrete derivatives:** Suppose, instead of a function, we start with a sequence $a_1, a_2, a_3, \ldots$ of numbers. Then the *discrete derivative* of this sequence is defined to be the sequence

   $$a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$$

   consisting of the consecutive differences of entries. For example, the discrete derivative of $1, 2, 3, 4, 5, \ldots$ is the sequence $1, 1, 1, 1, \ldots$.

(a) Find the discrete derivative of the sequence

$$3, 6, 9, 12, 15, \ldots.$$

In general, what kinds of sequences have discrete derivatives which are constant sequences?

(b) Find the discrete derivative of the sequence

$$1, 4, 9, 16, 25, \ldots$$

consisting of the square numbers n^2. Can you find a formula for the nth term of this sequence? How does this relate to the derivative of $f(x) = x^2$?

(c) Find the discrete derivative of the sequence whose nth term is $n^3 - n + 1$. How many times do we have to take the derivative of such a sequence before getting a constant sequence?

(d) Find the discrete derivative of the sequence

$$1, 2, 4, 8, 16, \ldots$$

whose nth term is 2^n. What about 3^n?

6. Consider the function f defined by

$$f(x) = \begin{cases} 0 & x \text{ is irrational} \\ 1 & x \text{ is rational} \end{cases}.$$

Is f differentiable at 0? How about $x \cdot f(x)$? How about $x^2 \cdot f(x)$?