CORRECTION TO "SEMISTABLE DEGENERATIONS AND PERIOD SPACES FOR POLARIZED K3 SURFACES"

MARTIN OLSSON

I am grateful to Alberto Bellardini for pointing out that the definition of the log Picard functor used in sections 3 and 4 of the paper is not correct when the singular locus is not connected. A corrected definition is as follows.

Let

$$(f, f^b) : (X, M_X) \to (S, M_S)$$

be a proper, special, log semistable morphism as in 4.1, which is cohomologically flat in dimension 0. Define $\mathscr{P}ic$ to be the fibered category over the category of S-schemes whose fiber over $T \to S$ is the groupoid of $M^{\mathrm{gp}}_{X_T}$-torsors P on X_T such that the associated $\overline{M}^{\mathrm{gp}}_{X_T}$-torsor $\overline{P} := P \times^{M^{\mathrm{gp}}_{X_T}} \overline{M}^{\mathrm{gp}}_{X_T}$ is étale locally on T trivial. With this definition theorem 4.4 then holds:

Theorem 1.1. *The fibered category $\mathscr{P}ic$ is an algebraic stack locally of finite type over S.*

Remark 1.2. Suppose that the singular locus of every geometric fiber of $f : X \to S$ is connected. This holds, in particular, for stable log K3 surfaces. Then the above definition of $\mathscr{P}ic$ agrees with the one given in the paper. Indeed in this case the log structure M_S defines a closed subscheme $S_0 \subset S$ and if $\nu : \widetilde{X}_0 \to X_0$ denotes the blowup of the restriction X_0 of X to S_0 as in the proof of 4.18, then $\overline{M}^{\mathrm{gp}}_X$ is isomorphic to $\nu_* \mathbb{Z}_{\widetilde{X}_0}$ and therefore $R^1 f_* \overline{M}^{\mathrm{gp}}_X = 0$ by the proper base change theorem and SGA 4, IX, 3.6. The same also remains true after arbitrary base change $T \to S$. It follows that for any M^{gp}_X-torsor P the associated $\overline{M}^{\mathrm{gp}}_X$-torsor $\overline{P}$ is étale locally on S trivial.

Remark 1.3. One can also consider the groupoid of pairs (P, ϵ), where P is an M^{gp}_X-torsor and ϵ is a trivialization of $\overline{P}$. This groupoid is equivalent to the groupoid of $\mathscr{O}^*_X$-torsors.

Paragraphs 4.9 through 4.15 of the proof of theorem 4.4 carries over as written for the above definition of $\mathscr{P}ic$. The only additional observations needed are the following:

(i) If P is an $M^{\mathrm{gp}}_{X_T}$ torsor on X_T for some T/S, then the condition that the $\overline{M}^{\mathrm{gp}}_{X_T}$-torsor $\overline{P}$ is trivial depends only on T_{red}, since $\overline{M}^{\mathrm{gp}}_{X_T}$ is a constructible sheaf.

(ii) In the verification that $\mathscr{P}ic$ is limit preserving one also has to appeal to SGA 4, VII, 5.7 to argue that the condition of local triviality of $\overline{P}$ is compatible with passing to the limit.

For the compatibility with completion, we show that proposition 4.17 holds with the corrected definition of $\mathscr{P}ic$. Proceeding as in the paper, strike the statement about H^1 in 4.18, and instead define Σ_n (resp. Σ) to be the kernel of the map

$$H^1(X_{\hat{A}_n}, M^{\mathrm{gp}}_{X_{\hat{A}_n}}) \to H^1(X_{\hat{A}_n}, \overline{M}^{\mathrm{gp}}_{X_{\hat{A}_n}}), \quad (\text{resp. } H^1(X_{\hat{A}}, M^{\mathrm{gp}}_{X_{\hat{A}}}) \to H^1(X_{\hat{A}}, \overline{M}^{\mathrm{gp}}_{X_{\hat{A}}})).$$

Then replace (4.20.2) with

$$\Sigma \to \varprojlim \Sigma_n,$$

and the two H^1's occurring in the end of the proof of 4.20 with Σ's.