ERRATA AND FURTHER COMMENTS – ALGEBRAIC SPACES AND STACKS

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I am grateful to Daniel Bergh, Antoine Chambert-Loir, Brian Conrad, and Minseon Shin for sending me corrections and comments.

General comment: The reader should be attentive to the fact that conventions differ in the literature about the properties of the diagonal in the definitions of algebraic spaces and stacks. The Stacks Project and [65] aside, one usually finds some standing assumptions about the diagonal such as it being quasi-compact.

p. 8, 14 lines from bottom. Replace "and" by "implies"

p. 9, proof of 1.1.6. Two instances where lower case v should be upper case V (so replace first v by V and second v_i by V_i).

p. 29, 1.5.3. In the definition of the functor $\underline{\mathrm{Quot}}^P(F/X/S)$ the sheaf G should also be assumed flat over S'.

p. 43. On the last line of the proof of 2.2.14 replace X by Y.

p. 86. There is a missing period at the end of exercise 3.E.

p. 124, Proposition 5.2.5. More generally it is true that the sheaf quotient of an étale equivalence relation in algebraic spaces is again an algebraic space. This result can be found in the Stacks Project (Tag 04S6) and with a quasi-compact assumption on the equivalence relation in [49, Proposition 1.3].

p. 130, Definition 5.4.5. For this definition of an imbedding to make sense we need to know that the property of being an imbedding is stable in the étale topology without any quasi-compactness assumptions. This result can be found in the Stacks Project Tag 02YM.

p. 134, Definition 5.4.15. It is not clear that the definition of a noetherian algebraic space given here is the "right" one. To avoid certain pathological behaviors one may want to impose also that a noetherian algebraic space be quasi-separated (which holds automatically for locally noetherian quasi-compact schemes). This is, for example, the definition used in the Stacks Project (see Tag 03E9).

p. 134, 5.5.1. The reference [49, A.4] proves that algebraic spaces are fpqc sheaves under the assumption that the diagonal is separated and of finite type. The result, without these assumptions on the diagonal, can be found in the Stacks Project as Tag 0APL, where it is attributed to Gabber.

p. 178, Definition 8.3.1. In the original definition of algebraic stack in [23, Definition 4.5] the diagonal is assumed representable by schemes, whereas in this definition of Deligne-Mumford stack the diagonal is assumed representable by algebraic spaces. To connect the two definitions, note that if $\mathscr{X}$ is a Deligne-Mumford stack in the sense of 8.3.1, and if furthermore the diagonal is assumed separated, then the diagonal is representable by schemes. Indeed in this case the diagonal is locally of finite type and formally unramified (8.3.3) and therefore locally quasi-finite which implies that it is representable by schemes (this requires a slight generalization of 7.2.10 to the case of locally quasi-finite algebraic spaces over a scheme which is Tag 0417 in the Stacks Project).

p. 179, Remark 8.3.4. In Tag 0DSN of the Stacks Project a result is shown generalizing this remark to remove the assumption that the diagonal is of finite presentation.

p. 182, Remark 8.3.6. Note that throughout [20] there is an assumption that diagonals are quasi-compact and separated.

p. 230, line 7. After "quasi-finite" there should be an end of proof symbol indicating the end of the proof of 11.2.6.

p. 233. The second citation in 11.4.1 should reference [70], not [71].

p. 247. In 12.2.5 the sequence displayed on the second line of the paragraph should be assumed central. Otherwise 12.2.6 (i) may fail.

p. 254. In the displayed equation in the proof of 12.3.11 the second $\mathscr{E}$ should be $\mathscr{A}$.

p. 261, third line. The reference should be to exercise 4.E, not 4.D.

p. 262. Replace the two instances of $[0 : 0 : 1]$ by $[0 : 1 : 0]$.

p. 292. Citation [36], the accent on "étales" is backwards.