Section 5.3

Evaluate the following integrals.

7) \( \int_{1}^{0} 3x - e^x \, dx \).

Hint: Let \( f(x) = x^2 - e^x \). Then \( f'(x) = 2x - e^x \) so
\( \int_{1}^{0} 2x - e^x \, dx = f(0) - f(1) = -2 + \frac{1}{e} \).

13) \( \int_{0}^{1} \left( \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x}} \right) \, dx \)

Note: \( x (\sqrt{x} + \sqrt{x}) = x^{3/2} + x^{1/2} \), which is the derivative of \( \frac{3}{2} x^{3/2} + \frac{1}{4} x^{1/2} \).

Thus \( \int_{0}^{1} \left( \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x}} \right) \, dx = \frac{3}{2} (1) + \frac{1}{4} (1) \frac{1}{3} = 55/63 \).

29) What is wrong with \( \int_{1}^{3} \frac{1}{x^2} \, dx = \left[ \frac{x^{-1}}{-1} \right]_{1}^{3} = -4/3 \).

\( \frac{1}{x^2} \) is not continuous on \( [1, 3] \) so the evaluation theorem does not apply.

35) Evaluate \( \int_{0}^{1} x^3 \, dx \), interpret it as an area and sketch the graph.

Note: \( \int_{0}^{1} x^3 \, dx \, dx = \left[ \frac{x^4}{4} \right]_{0}^{1} = \frac{1}{4} - \frac{1}{4} = 15/4 \). This is the area under \( x^3 \) from 0 to 1 minus the area over the curve from -1 to 0.
38) Verify that \( \int x \cos(x) \, dx = x \sin(x) + \cos(x) + C \).

Differentiating, we get \( \frac{d}{dx}(x \sin(x) + \cos(x) + C) = x \cos(x) - \sin(x) \)

which verifies the formula.

41) Find the general indefinite integral: \( \int (1-t)(2+t^2) \, dt \)

Note \((1-t)(2+t^2) = 2 + t^2 + (-2t) + (-t^3) = 2 - 2t + t^2 - t^3\),

which is the derivative of \(2t - \frac{1}{3}t^3 - \frac{1}{4}t^4 + C\).

Thus \( \int (1-t)(2+t^2) \, dt = 2t - \frac{1}{2}t^2 - \frac{1}{4}t^4 + C \).

45) Evaluate \( \int_{0}^{2} 2y - y^2 \, dy \).

\[
\int_{0}^{2} 2y - y^2 \, dy = \left[ y^2 - \frac{1}{3}y^3 \right] \bigg|_{0}^{2} = 4 - \frac{8}{3} = 4/3.
\]

55) The velocity of a particle is \( v(t) = 3t - 5 \), \( 0 \leq t \leq 3 \), in \( \text{m/s} \). Find (a) the displacement and (b) the distance traveled by the particle.

(a) The displacement is \( \int_{0}^{3} 3t - 5 \, dt = \left[ \frac{3}{2}t^2 - 5t \right]_{0}^{3} = -3/2 \text{m} \).

(b) The distance is \( \int_{0}^{3} |3t - 5| \, dt = \int_{0}^{5/3} 3t - 5 \, dt + \int_{5/3}^{3} 5 - 3t \, dt \)

\[
= \left[ \frac{3}{2}t^2 - 5t \right]_{0}^{5/3} + \left[ 5t - \frac{3}{2}t^2 \right]_{5/3}^{3} \\
= \frac{25}{6} + \frac{8}{3} = 41/6 \text{ m}.
\]
61) Water flows out of a tank at \( r(t) = 200 - 4t \) liters per minute, where \( 0 \leq t \leq 50 \). How much water flows from the tank in the first 10 minutes?

The amount of water is

\[
\int_{0}^{10} (200 - 4t) \, dt = \left[ 200t - 2t^2 \right]_{0}^{10} = 1800 \text{ L}.
\]