section 2.3

19. Since \( y = x^{3/2} + 4x^{1/2} + 3x^{-1/2} \), we have

\[
y' = (x^{3/2})' + 4(x^{1/2})' + 3(x^{-1/2})' = \frac{3}{2}x^{1/2} + 4 \cdot \frac{1}{2}x^{-1/2} + 3 \left( -\frac{1}{2} \right) x^{-3/2}
\]

\[
= \frac{3}{2}x^{1/2} + 2x^{-1/2} - \frac{3}{2}x^{-3/2}.
\]

30. 
\[
G'(r) = (r^{1/2})' + (r^{1/3})' = \frac{1}{2}r^{-1/2} + \frac{1}{3}r^{-2/3}
\]
and
\[
G''(r) = \frac{1}{2}(r^{-1/2})' + \frac{1}{3}(r^{-2/3})' = \frac{1}{2} \left( -\frac{1}{2} \right) r^{-3/2} + \frac{1}{3} \left( -\frac{2}{3} \right) r^{-5/3} = -\frac{1}{4}r^{-3/2} - \frac{2}{9}r^{-5/3}.
\]

33. We know that
\[
\frac{d}{dx} \sin(x) = \cos(x),
\]
\[
\frac{d^2}{dx^2} \sin(x) = \frac{d}{dx} \cos(x) = -\sin(x),
\]
\[
\frac{d^3}{dx^3} \sin(x) = \frac{d}{dx} (-\sin(x)) = -\cos(x),
\]
\[
\frac{d^4}{dx^4} \sin(x) = \frac{d}{dx} (-\cos(x)) = \sin(x),
\]
and we get \( \sin(x) \) back at the 4th derivative. If we keep differentiating, we will get the 4 functions above periodically. So for any integer \( k \geq 0 \), we have
\[
\frac{d^{4k+1}}{dx^{4k+1}} \sin(x) = \cos(x),
\]
\[
\frac{d^{4k+2}}{dx^{4k+2}} \sin(x) = -\sin(x),
\]
\[
\frac{d^{4k+3}}{dx^{4k+3}} \sin(x) = -\cos(x),
\]
\[
\frac{d^{4k}}{dx^{4k}} \sin(x) = \sin(x).
\]
Since 99 = 4\( k + 3 \) for some integer \( k \), we have
\[
\frac{d^{99}}{dx^{99}} \sin(x) = -\cos(x).
\]

35. If the tangent line at \( x = a \) is horizontal, the tangent slope \( f'(a) = 0 \). So we should solve \( f'(x) = 0 \).

\[
f'(x) = x' + 2(\sin(x))' = 1 + 2 \cos(x) = 0, \quad \cos(x) = -\frac{1}{2},
\]
so \( x = 2k\pi + \frac{2\pi}{3} \) for \( k \) ranging over all integers.

41. (a) By definition, the velocity \( v(t) \) is the derivative of \( s(t) \), and the acceleration \( a(t) \) is the derivative of \( v(t) \). So

\[
v(t) = s'(t) = (t^3)' - 3t' = 3t^2 - 3
\]
and
\[ a(t) = v'(t) = 3(t^2)' - 3' = 6t - 0 = 6t. \]

(b) \( a(2) = 12. \)

(c) This means we should evaluate \( a(t) \) at the solutions of \( v(t) = 0 \), which is \( t = 1 \), so \( a(1) = 6. \)

51. The rate of increase, by definition, is the derivative of \( S(r) \), so it’s \( S'(r) = 4\pi(r^2)' = 8\pi r. \)

\[ S'(1) = 8\pi, \quad S'(2) = 16\pi, \quad S'(3) = 24\pi. \]

58. If \( y = Ax^2 + Bx + C \), then \( y' = 2Ax + B \) and \( y'' = 2A. \) Substitute them into the differential equation we have

\[ 2A + 2Ax + B - 2(Ax^2 + Bx + C) = x^2, \]

which is

\[ -2Ax^2 + (2A - 2B)x + (2A + B - 2C) = x^2. \]

This holds for any \( x \), so the corresponding coefficients of the two sides are equal, and we have

\[
\begin{align*}
-2A &= 1 \\
2A - 2B &= 0 \\
2A + B - 2C &= 0,
\end{align*}
\]

and solve it we get

\[
\begin{align*}
A &= -\frac{1}{2} \\
B &= -\frac{1}{2} \\
C &= -\frac{3}{4}.
\end{align*}
\]

61. The tangent slope at \( x = 2 \) is \( y'(2) = 2ax|_{x=2} = 4a \), and the line \( 2x + y = b \) can be rewritten as \( y = -2x + b \), so the slope is \(-2\). If this line is the tangent line to the parabola at \( x = 2 \), they must have the same slope, so

\[ 4a = -2, \quad a = -\frac{1}{2}. \]

The \( y \)-coordinate of the point is \( y = ax^2 = -\frac{1}{2} \cdot 2^2 = -2. \) The line \( 2x + y = b \) is tangent to the parabola at the point \((2, -2)\), so the point lies on the line, and we have

\[ 2 \times 2 - 2 = b, \quad b = 2. \]

So \( a = -1/2 \) and \( b = 2. \)