

## MATH 748: HOMEWORK 6

- (1) Milne 4-2
- (2) Milne 4-3
- (3) Let  $K$  be a quadratic field and  $I$  an ideal of  $\mathcal{O}_K$ . Show that  $I = (a, \beta)$  for some  $a \in \mathbb{Z}$  and  $\beta \in \mathcal{O}_K$ . Show that  $(a, \beta)(a, \bar{\beta})$  is principal, where  $\bar{\beta}$  is the conjugate of  $\beta$ .
- (4) For an ideal in  $\mathcal{O}_K$ , let  $\phi(I) = |(\mathcal{O}_K/I)^*|$ . Show that  $\phi(I) = \text{Nm}(I) \prod_{\mathfrak{p}|I} (1 - \frac{1}{\text{Nm } \mathfrak{p}})$ , where the product is over prime ideals of  $\mathcal{O}_K$ .
- (5) In class we showed that a prime ramifies in  $\mathcal{O}_K$  if and only if it divides the discriminant. Use Dedekind's criterion to give another proof of this fact when  $\mathcal{O}_K = \mathbb{Z}[\alpha]$  (e.g. when  $K$  is quadratic).