MATH 748: HOMEWORK 2 (FINAL)

(1) Milne 2-1, 2-3, **2-6 (added!)**
(2) Let $K \subset L \subset M$ be separable extensions of fields. Show that $\mathrm{Tr}_{M/K} = \mathrm{Tr}_{L/K} \circ \mathrm{Tr}_{M/L}$ (a precisely similar argument shows that $\mathrm{Nm}_{M/K} = \mathrm{Nm}_{L/K} \circ \mathrm{Nm}_{M/L}$, but you only need to do the trace version).
(3) Let $d > 1$ be an integer. Show that the only units in the ring $\mathbb{Z}[\sqrt{-d}]$ are ± 1.
(4) Let $K = \mathbb{Q}(\alpha)$ where α is a root of $f(x) = x^5 + 2x^4 + 5x^3 + 3x^2 + 4$. Use sage to check that this polynomial is irreducible. Use sage to compute $\mathrm{Disc}(\mathcal{O}_K/\mathbb{Z})$.
(5) Find a domain A and an element b in some domain B containing A such that b is integral over A but the minimal monic polynomial of B over the fraction field of A does not have coefficients in A.
(6) For $K = \mathbb{Q}(\sqrt{3})$ and $K = \mathbb{Q}(\sqrt{5})$, state the integral basis for $\mathcal{O}_K$ you found in the last homework set and find a dual basis with respect to the trace form for it.