MATH 748: HOMEWORK 10

(1) Milne 6,1
(2) Milne 6-2
(3) Let p be an odd prime. Compute the discriminant of $(X^p - 1)/(X - 1)$. Deduce that $\mathbb{Q}(\zeta_p)$ (ζ_p a primitive pth ROU) contains a quadratic field with discriminant $\pm p$. Show using the Minkowski bound that $\mathbb{Z}[\zeta_p]$ is a UFD for $p = 5, 7$.
(4) Let $K = \mathbb{Q}(\alpha)$, where α is a root of $x^5 + 2x^4 + 5x^3 + 3x^2 + 4$. Use sage to compute the ring of integers of K, the discriminant, the class group (including generators and relations), the units in the ring of integers (including fundamental units, and don't forget the torsion part) and express $-(43519/2)\alpha^4 + 23722\alpha^3 + (52935/2)\alpha^2 - (59495/2)\alpha + 32122$ in terms of fundamental units and torsion.
(5) Let $K_1 = \mathbb{Q}(\sqrt{d_1})$, $K_2 = \mathbb{Q}(\sqrt{d_2})$ be distinct real quadratic fields. Define $K_3 = \mathbb{Q}(\sqrt{d_1 d_2})$ and $K = K_1 K_2$. Show that $[\mathcal{O}_K^* : \mathcal{O}_{K_1}^* \mathcal{O}_{K_2}^* \mathcal{O}_{K_3}^*] \leq 8$. Using sage, try a reasonable number of cases so as to guess what values of $[\mathcal{O}_K^* : \mathcal{O}_{K_1}^* \mathcal{O}_{K_2}^* \mathcal{O}_{K_3}^*]$ actually occur, and make a conjecture.