MATH 748: HOMEWORK 1 (FINAL)

(1) Milne 2-2, 2-4
(2) In the polynomial ring $A = \mathbb{Q}[x, y]$, consider the ideal $\wp = (x^2 - y^3)$. Show that $\wp$ is a prime ideal, but that $A/\wp$ is not integrally closed.
(3) Find the minimal polynomials over $\mathbb{Q}$ of $(1 + i)\sqrt{3}$ and $i + \sqrt{3}$.
(4) Is $\frac{3+2\sqrt{6}}{1-\sqrt{6}}$ an algebraic integer? What about $\frac{\sqrt{5}}{\sqrt{2}}$? $\frac{\sqrt{3}+\sqrt{7}}{2}$? $\frac{1+\sqrt[3]{10}+\sqrt[3]{100}}{3}$?
(5) Show that the quadratic (i.e. degree 2) extensions of $\mathbb{Q}$ are exactly $\mathbb{Q}(\sqrt{D})$ for square-free integers $D \neq 0, 1$,
(6) For every square-free integer $D \neq 0, 1$, find the ring of algebraic integers in $\mathbb{Q}(\sqrt{D})$.
(7) Use sage to find the minimal polynomial of $\frac{(\sqrt[4]{2})^3}{2} + \frac{\sqrt[4]{2}}{2} + 1$ over $\mathbb{Q}$. Is it an algebraic integer?