Midterm 2

Due Tuesday, April 18 at 10pm with no exceptions. Please upload a legible pdf to Gradescope.

You may NOT work together. You may use your notes, the books, and any material on the class webpage, but nothing else- no online resources, no software (exept for typsetting), and no discussion with anyone else. Show all work and justify all answers. Point values are marked, for a total of 32 points.

1. Find $\int_C f(z)dz$ where $f(z) = \frac{\cos 2z}{(z-i-1)^3}$ and C is

   a) (3 points) A circle of radius 2 centered at 0, positively oriented.

   b) (3 points) The suare formed by the lines $x = -1$, $x = 0$, $y = 0$, $y = 1$, positively oriented.

2. (4 points) Let $f(z) = \text{Log}(1 - \frac{1}{z})$. Prove that

$$\int_{C_1} f(z)dz = \int_{C_2} f(z)dz$$

   where C_1 is the circle centered at 0 with radius 4 and C_2 is the square formed by the lines $x = -2$, $x = 2$, $y = -2$, $y = 2$, both positively oriented.

3. Find the Laurent expansion centered at 2 of $f(z) = \frac{z}{(z-2)(z-2-i)}$

   a) (3 points) For z such that $0 < |z - 2| < 1$

   b) (3 points) For z such that $1 < |z - 2|$

   c) (4 points) Find $\text{Res}_{z=2} f(z)$ and $\text{Res}_{z=\infty} f(z)$.

4. Let $f(z) = \frac{\sin(z)}{z^3+z}$.

   a) (3 points) Find the singularities of f. Identify whether each is a removable singularity or a pole of order m. For the poles, find m.

   b) (3 points) Compute the residue of f at each pole.

   c) (2 points) Compute $\int_C f(z)dz$, where C is a circle of radius 3/2 centerd at $2i$.

5. (4 points) Let f be analytic on and interior to C_R, the circle of radius $R > 0$ centered at 0. Suppose that $|f(z)| \leq M$ for all z such that $|z| = R$. Use the Cauchy Integral Formula to prove that

$$|f'(z_0)| \leq \frac{MR}{(R - |z_0|)^2}$$

   for all z_0 such that $|z_0| < R$.

At the end of your exam solutions, please copy the sentence "I did not discuss this exam with anyone and I did not use any resources except the textbook, notes, homework, and course documents." and sign your name.