Midterm 1

Due Tuesday, March 7 at 10pm with no exceptions. Please upload a legible pdf to Gradescope.

You may NOT work together. You may use your notes, the books, and any material on the class webpage, but nothing else- no online resources, no software, and no discussion with anyone else. Show all work and justify all answers. Each question is worth 6 points, for a total of 30 points.

1. For each function describe the set of $z \in \mathbb{C}$ for which the function is differentiable, give the derivative at each of those points, and describe the set of z at which the function is analytic. Justify all answers completely.

   a) $f(z) = e^{-y}\left[(x\cos(x) - y\sin(x)) + i(x\sin(x) + y\cos(x))\right]$

   b) $g(z) = \bar{z}^2 + i\bar{z}$

2. Let $f(z)$ be an analytic function defined on a domain with image contained in

   (a) the line through $x_0 \in \mathbb{R}$ making an angle of θ_0 with the $x-$axis

   (b) the circle centered at z_0 of radius R

   In each case, prove that f is constant.

3. Define an analytic branch $f(z)$ of z^i such that $f(i) = e^{3\pi/2}$, $f(-1) = e^{\pi}$. Give the domain of $f(z)$ and compute $f(-1+i)$. Justify all computations; there are multiple correct solutions.

4. Let $f(z) = e^z + \bar{z}^2$ and let C be the triangle with vertices 0, i, and $-1+i$ oriented counterclockwise. Compute $\int_C f(z)dz$.

5. Prove that $f(z) = \frac{3z}{z-2}$ does not have an antiderivative on the domain $D = \mathbb{C}\backslash\{2\}$ (i.e. the complex plane with the point 2 removed.) Hint: use a contour integral.

At the end of your exam solutions, please copy the sentence "I did not discuss this exam with anyone and I did not use any resources except the textbook, notes, homework, and course documents." and sign your name.