$\boxed{\S 5}$

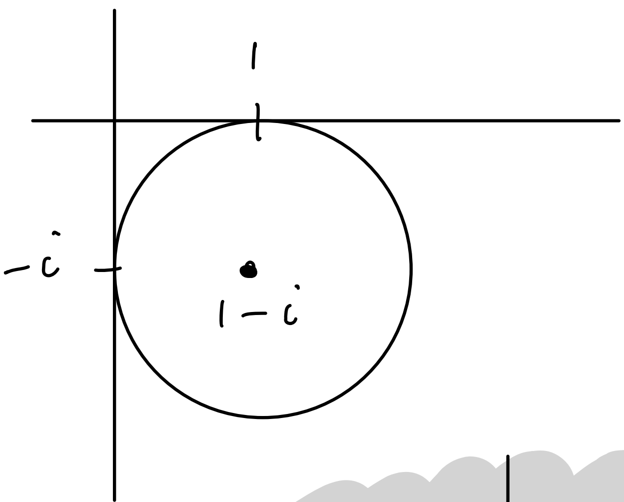
4) $$\sqrt{2}\sqrt{x^2+y^2} \geq |x|+|y|$$ $\Updownarrow$ $\sqrt{\cdot}$ preserves $\geq$ on $\mathbb{R}_{\geq 0}$

$$2(x^2+y^2) \geq x^2+y^2+2|x||y|$$ $\Updownarrow$

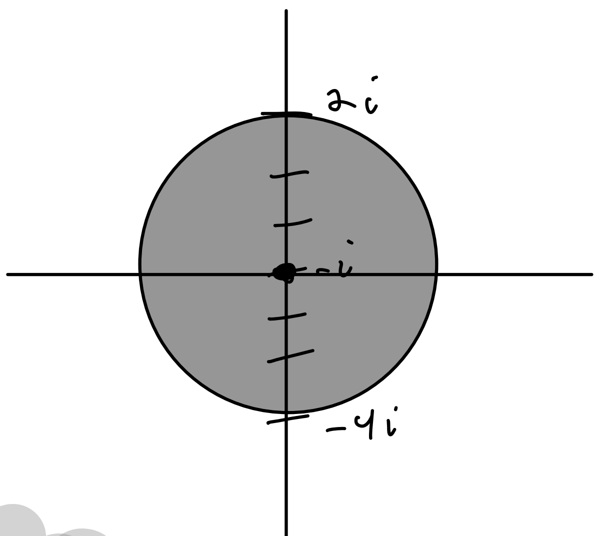
$$x^2+y^2-2|x||y| \geq 0$$ $\Updownarrow$

$$(|x|-|y|)^2 \geq 0$$ $\Updownarrow$

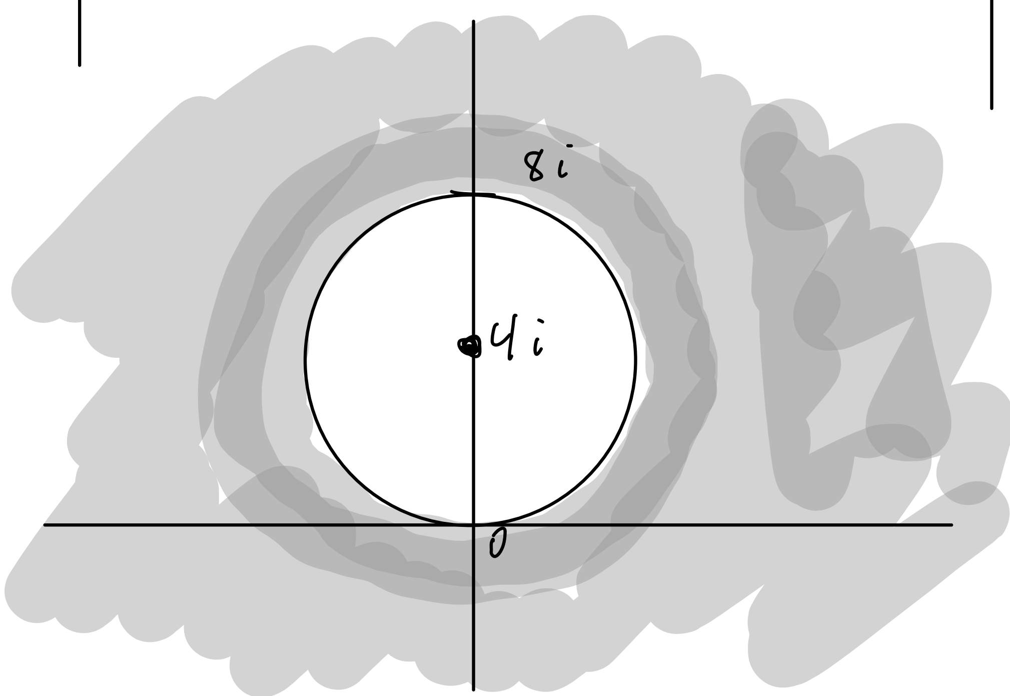
5) a) $|z-(1-i)|=1$



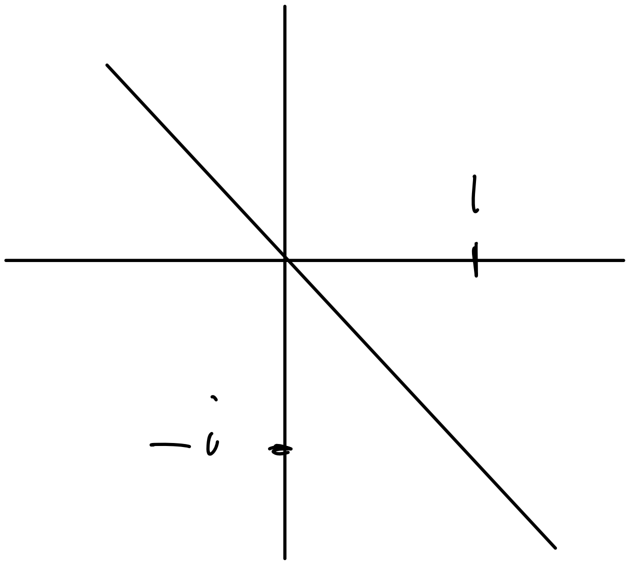
b) $|z-(-i)| \leq 3$



c) $|z-4i| \geq 4$



6)



points equidistant from 1 and $-i$

7) choose R such that

$$R^{n-i} > \frac{n|a_i|}{|a_n|}$$

for $i=0,1,\dots,n-1$. Then if $|z|>R$,

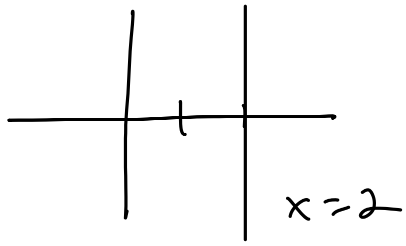
$$\frac{|a_i|}{|z|^{n-i}} < \frac{|a_n|}{n},$$ so by (9) on p. 13

$|w| \le |a_n|$ and

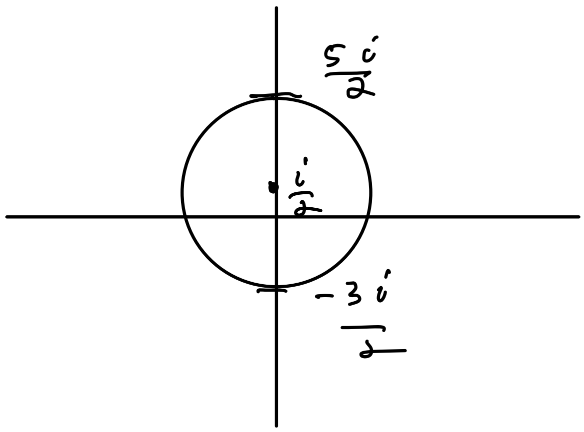
$|P(z)| = |a_n+w||z|^n \le 2|a_n||z|^n.$

§6

2) a) $Re(\bar{z} - i) = Re(x + i(-y-1)) = x$



b) $|2\bar{z} + i| = |\overline{2z - i}| = |2z - i| = 2|z - \frac{i}{2}|$



7) $|Re(2 + \bar{z} + z^3)| \le |2 + \bar{z} + z^3|$

$\le 2 + |\bar{z}| + |z|^3 = 2 + |z| + |z|^3 \le 2 + 1 + 1^3 = 4$

9) $|z^4 - 4z^2 + 3| = |(z^2-3)(z^2-1)| \ge$

$||z|^2 - |-3||\,||z|^2 - |-1|| = |4-3||4-1| = 3$

$\left|\frac{1}{z^4 - 4z^2 + 3}\right| = \frac{1}{|z^4 - 4z^2 + 3|} \le \frac{1}{3}$.

14) $1 = \left(\frac{z+\bar{z}}{2}\right)^2 - \left(\frac{z-\bar{z}}{2i}\right)^2$

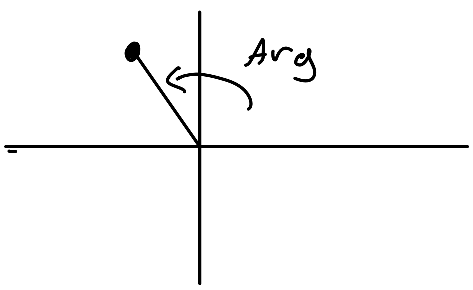
$$= \frac{1}{4}\left(z^2 + 2z\bar{z} + \bar{z}^2 + z^2 - 2z\bar{z} + \bar{z}^2\right)$$

$$= \frac{z^2+\bar{z}^2}{2}$$

$\boxed{\S 9}$

1) a) $\frac{-2}{1+\sqrt{3}i}\frac{(1-\sqrt{3}i)}{(1-\sqrt{3}i)} = \frac{-2+2\sqrt{3}i}{4} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\text{Arg} = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$

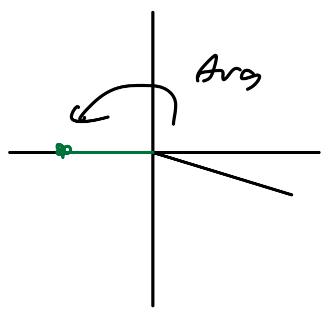


b) $\sqrt{3} - i = 2\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$

$$= 2e^{i\left(-\frac{\pi}{6}\right)}$$

$$(\sqrt{3}-i)^6 = 2^6 e^{-i\pi} = 2^6 e^{i\pi}$$

$\text{Arg} = \pi$



6) Since $\text{Re}(z_1), \text{Re}(z_2) > 0$, $\text{Arg}(z_1), \text{Arg}(z_2) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\text{Arg}(z_1) + \text{Arg}(z_2) \in (-\pi, \pi)$.

Thus $z_1 z_2 = r_1 r_2 e^{i(\text{Arg}(z_1) + \text{Arg}(z_2))}$

and $\text{Arg}(z_1) + \text{Arg}(z_2)$ is a value of $\arg(z_1 z_2)$.

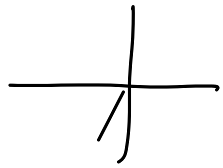
Since it is in the range $(-\pi, \pi]$,

$$\mathrm{Arg}(z_1 z_2) = \mathrm{Arg}(z_1) + \mathrm{Arg}(z_2).$$

$\boxed{\S 11}$

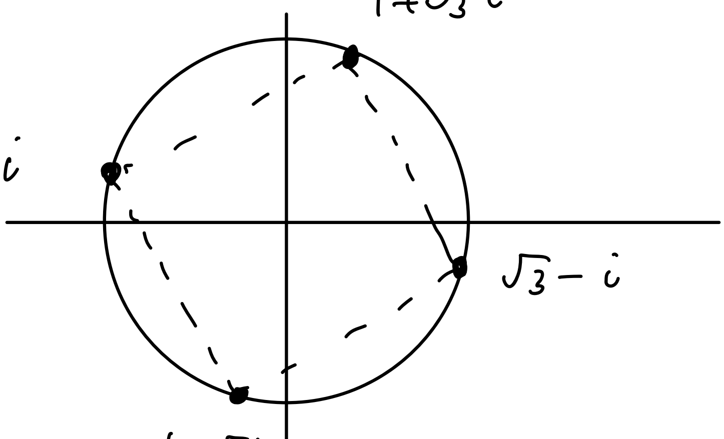
3) $$(-8-8\sqrt{3}\, i) = 16\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$$

$$= 16\, e^{-\frac{2\pi}{3} i}$$



$$(-8-8\sqrt{3} i)^{1/4} = 2\left(e^{-i\frac{\pi}{6}}, e^{-i\frac{\pi}{6}+i\frac{\pi}{2}}, e^{-i\frac{\pi}{6}+i\pi}, e^{-i\frac{\pi}{6}+i\frac{3\pi}{2}}\right)$$

$$= 2e^{-i\frac{\pi}{6}},\ 2e^{i\frac{\pi}{3}},\ 2e^{i\frac{5\pi}{6}},\ e^{-i\frac{2}{3}\pi}$$



Since $\mathrm{Arg}(-8-8\sqrt{3} i) = -\frac{2\pi}{3}$, $2e^{-i\frac{\pi}{6}}$ is the principal value.

6) $(-4)^{1/4} = (1+i)(1, i, -1, -i)$

$= 1+i, \ -1+i, \ -1-i, \ 1-i$

$(z-(1+i))(z-(1-i)) = z^2 - 2z + 2$

$(z-(-1+i))(z-(-1-i)) = z^2 + 2z + 2$

7) $c(1 + c + \dots + c^{n-2} + c^{n-1})$

$= c + c^2 + \dots + c^{n-1} + c^n$

$= c + c^2 + \dots + c^{n-1} + 1$

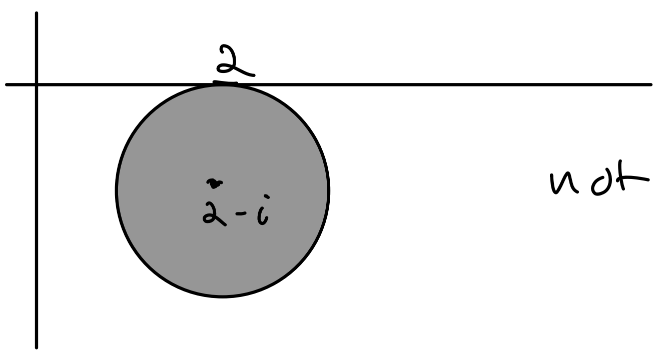
same

So $(c-1)(1 + c + c^2 + \dots + c^{n-1}) = 0.$

If $c \neq 1$, $1 + c + \dots + c^{n-1} = 0.$

$\boxed{\S\, 12}$

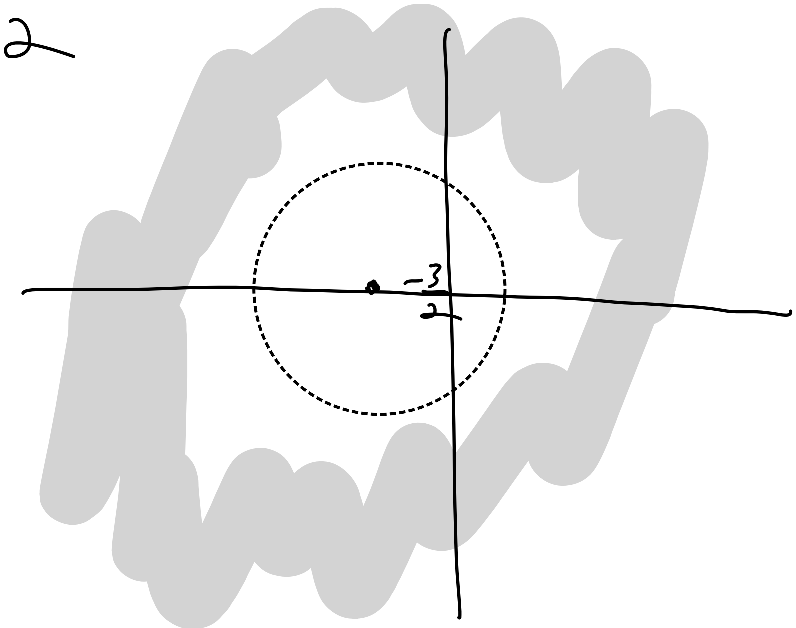
1) a)



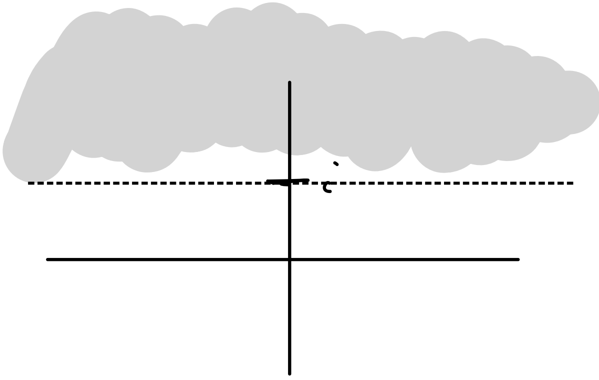
not open

b) $|2-(-\frac{3}{2})| > 2$

open + connected

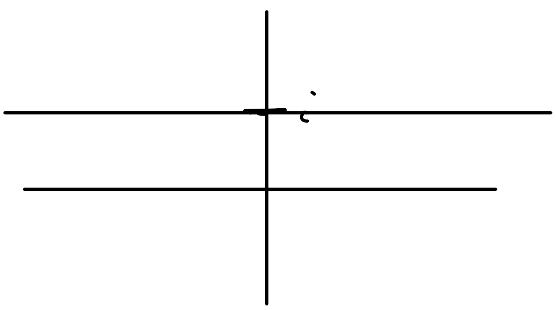


c)



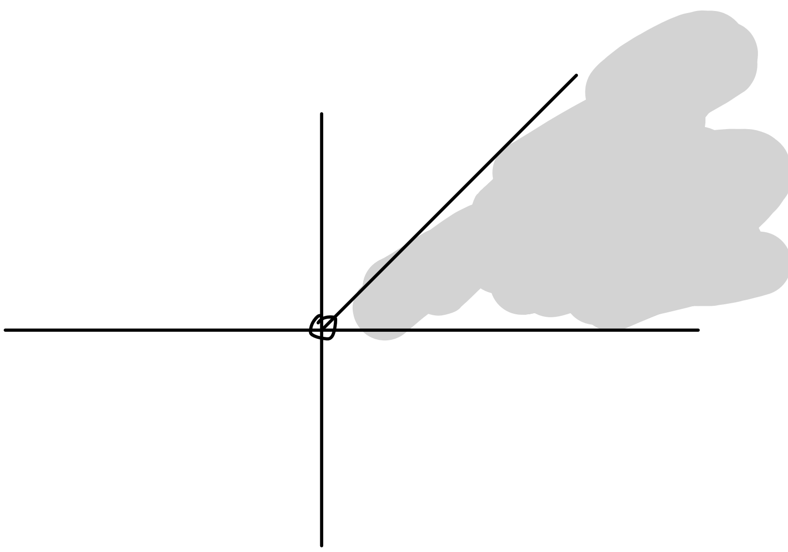
open + connected

d)



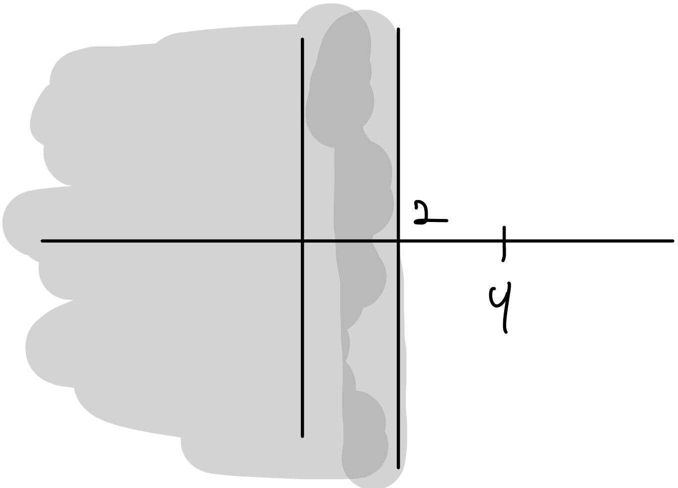
not open

e)



not open

f)



distance to 4 $\geq$

distance to 0

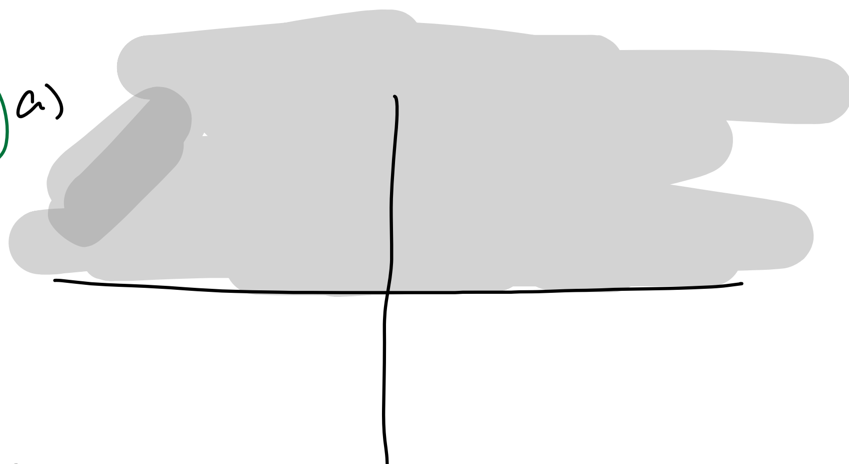
not open

(algebraically ; $(x-4)^2+y^2 \geq x^2+y^2$

$$-8x+16 \geq 0$$

$$x \leq 2$$

4) a)



add boundary points:

real axis

b) This is always true; $\mathbb{C}$

c) $\frac{x}{x^2+y^2} \leq \frac{1}{2} \Rightarrow x^2+y^2-2x \geq 0$

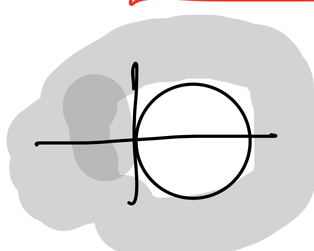
$$\Rightarrow x^2-2x+1+y^2 \geq 1$$

$$\Rightarrow (x-1)^2+y^2 \geq 1$$

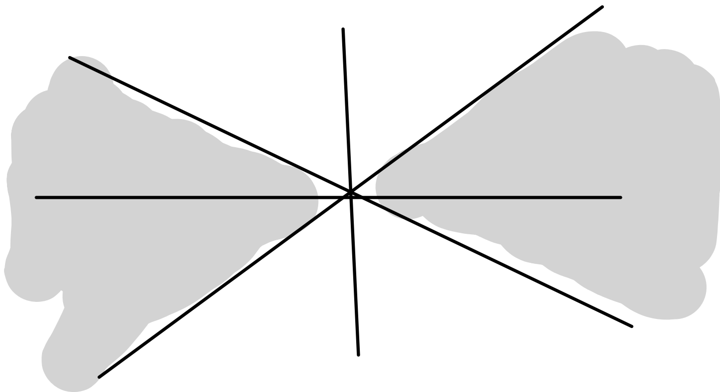
$$\Rightarrow |z-1| \geq 1$$

already closed

complete the square

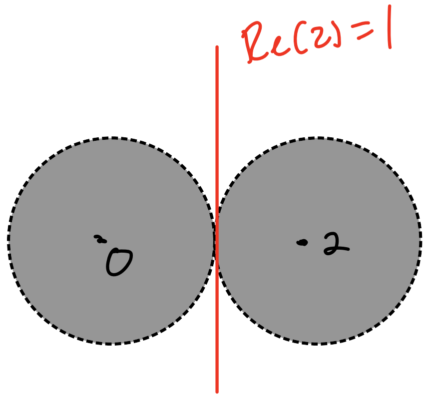


d) $x^2 - y^2 > 0 \implies |x| > |y|$



add boundary $|x| = |y|$

5)



At some point z along a polygonal line from 0 to 2, $Re(z) = 1$; but then

$|z| \geq Re(z) = 1$ and $|z-2| \geq Re(2-z) = 2 - Re(z) = 1$

so $z \notin S$.